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Effects of Right-Handed Neutrinos dipoles on Big Bang Nucleosynthesis

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Abstract

The Standard Model of particle physics provides an incredibly accurate description of fundamental interactions, yet it fails to account for the experimentally observed non-vanishing neutrino masses. The Type-I Seesaw mechanism offers an elegant solution to this problem by introducing right-handed, completely sterile neutrinos (Heavy Neutral Leptons, or HNLs). Since the full high-energy theory remains unknown, this thesis investigates the extended phenomenology of these sterile states using a model-independent approach based on the Standard Model Effective Field Theory (SMEFT) framework. We specifically focus on the effects of the dimension-five sterile dipole operator ($\mathcal{O}_{NB}^{(5)}$) for sterile neutrino masses in the range 0.1 GeV $\leqslant M_N \leqslant$ 10 GeV. This higher-dimensional operator couples the HNLs to the hypercharge field strength tensor, opening novel radiative decay channels such as $N_2 \to N_1\gamma$ and $N_i \to \nu_\alpha\gamma$. In this work, we compute the total decay widths of the sterile states, comparing the dipole-induced radiative decays with the standard processes induced solely by active-sterile mixing. Crucially, the lifetime of these particles is subject to stringent cosmological bounds: to preserve the successful predictions of Big Bang Nucleosynthesis (BBN) regarding the primordial yields of light nuclei, sterile neutrinos must decay before the weak interactions freeze-out ($\tau_i \leqslant 1$ s). By mapping this cosmological constraint onto the mass-mixing parameter space, we show that the inclusion of the dipole interaction may significantly enhances the decay rates, effectively shrinking the BBN-excluded regions. This relaxation allows for active-sterile mixing angles substantially smaller than those permitted in a purely mixing-driven scenario. Furthermore, we analyze how a non-vanishing mass splitting between the sterile states activates sterile-sterile transitions, introducing non-trivial modifications to these cosmological bounds.

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Chapter 1

Introduction

The formulation of the Standard Model (SM) of particle physics has been a great triumph for theoretical physics. Although the model is able to predict very accurately the behavior and properties of almost every particle we have been able to detect up to now together with many observed phenomena, it is far from being complete. It presents, in fact, major flaws when it comes to explaining certain phenomena and from the discrepancies between its prediction and observational data the need to go Beyond the SM and look for New Physics (NP) arises. The first source of those discrepancies was neutrino oscillations, which imply that these particles are not massless, followed then by cosmological observations revealing deeper issues. Research on galaxy rotational velocities, type Ia supernovae luminosity and galaxy clusters, like the Bullet Cluster, through gravitational weak lensing, revealed that baryonic matter[1] corresponds only to 5% of the Universe, while of the remaining content around 27% is made by dark matter and the last 68% is known as dark energy[1, 2, 3]. Concerning dark matter, from those observations we expect it to be made by massive, slow and weakly interacting particles, properties that exclude all the particle content of the SM from the candidates list. Dark energy, instead, appears to be the cause for the current expansion of the Universe, challenging once again the SM for a solution. The list of motivations for going BSM goes on with the baryon asymmetry and inflation. The first points out that the equations describing the primordial plasma are symmetric with respect to matter and antimatter production, requiring the Universe to annihilate into photons at the start of its life, unless the model provides a CP symmetry violation greater than the SM one[4]. Lastly, inflation answers the horizon problem very well, explaining why the CMB survey shows an isotropic Universe at scales much larger than what is expected from the area connected by causality, but it would require a new scalar field with a potential much different from the Higgs boson, challenging once again the SM particle content[5].

It comes natural, then, to look for New Physics in the lepton sector and, in particular, in the neutrino sector. In fact, the generation of extremely light neutrino masses is required to meet the observations from neutrino experiments like SuperKamiokande [6]. One elegant solution to solve this problem is provided by the Type-I Seesaw mechanism, that postulates the introduction of right-

[1] In this context this term is used to refer to matter and particles we can interact with and we know up to now.

handed neutrinos. Although these new states are defined as singlets under the SM gauge group $G_{SM} = SU(3)_c \times SU(2)_L \times U(1)_Y$, they can interact with the rest of the SM particle content through the active neutrinos. The Seesaw model, indeed, allows for a small mixing between the new (sterile) neutrinos and the already present (active) ones. This mixing has the double effect of allowing Neutral and Charged Current interactions for the RH fields and, notably, to generate a small but non-vanishing mass for active neutrinos. It is interesting to notice that the Type-I Seesaw model can provide a solution not only to the neutrino mass problem, but can also solve baryogenesis and provide a viable dark matter candidate [7]. All these discrepancies can even be solved at the same time if the right configuration for the sterile field are chosen, but this goes beyond the scope of the present discussion.

Interactions between sterile neutrinos and the SM are not provided only by the mixing. In a low-energy theory, irrelevant, higher-dimensional operators must be taken into account for a complete description of the phenomenology. Their presence in the theory is caused by unknown interactions and heavy fields present only at extremely high energies, but capable of generating new vertices in the Infrared (IR) low-energy Lagrangian. Those fields live at inaccessible mass scales Λ and must therefore be *integrated out* from the action when working at $E \ll \Lambda$, generating additional effective operators, suppressed by inverse powers of the cutoff scale Λ [8, 9]. Less suppressed operators generically give non-negligible contributions to the phenomenology. The most interesting among them for the sterile neutrino physics is the dipole operator ($\mathcal{O}^{(5)}_{NB}$) that, as we are going to discuss, will open a new channel for neutrino interactions and decays [10, 11]. The new radiative portal adds a correction to the total decay width comparable or even greater than the active-sterile mixing contribution, depending on the relative coefficients. This becomes important in determining the total decay width and, consequentially, the sterile neutrino lifetime.

The presence of these right-handed neutral leptons must be considered also in the Early Universe since they may lead to profound alterations of the physics. In particular, changes in the energy density of the primordial plasma can constitute a problem since they would affect theoretical predictions, spoiling their agreement with observational data. The Big Bang Nucleosynthesis (BBN) predictions are the most sensitive ones to the introduction of sterile neutrinos in the theory. A stringent bound on their lifetime is therefore required to avoid any unintended new contribution.

This thesis is structured as follows. In Chapter 2 the Type-I Seesaw model will be motivated starting from neutrino oscillation experiments. Then, the Minimal Seesaw Model will be introduced and studied, showing how two sterile neutrinos are sufficient for explaining the active neutrino mass generation [14]. In Chapter 3 the Effective Field Theory will be discussed and the νSMEFT framework defined. Then the dimension-five sterile dipole operator will be explored together with the radiative decays it induces. Finally, Chapter 4 presents the core analysis of the thesis: the evaluation of the BBN bounds on the sterile neutrino parameter space, demonstrating how the interplay between mixing- and dipole-induced decays shapes the cosmologically viable regions for New Physics. Finally, we discuss in Appendix A the mixing-induced decay widths and show explicitly the computation of one decay channel.

Chapter 2

Neutrino Physics in the Seesaw Mechanism

2.1 Neutrino oscillation experiments

Neutrino oscillation experiments such as Super-Kamiokande [6], have detected light but massive neutrinos that oscillate between their flavor as they travel, instead of massless ones, and provide most of the knowledge we have about them. Their setups consist of a source producing neutrinos of a specific flavor (e, μ or τ)[1], for example through β decay, and an extremely sensitive detector, placed at a distance L from one another. The surprising result obtained consists of the detection of neutrino flavors distinct from those originally produced. Moreover, in those experiments using two detectors at different distances, different neutrino flavors are detected even between them, implying a position dependence in the flavor composition of the neutrino beam, as if it oscillates during the particle travel. The "disappearance" and "appearance" of new neutrinos require the particle states described in the flavor basis to be related with their states in the mass basis (corresponding to the physical fields that enter in the SM Lagrangian) in this way:

$$\begin{aligned} |\nu_\alpha\rangle &= U^*_{\alpha i}|\nu_i\rangle, \\ |\nu_i\rangle &= U_{\alpha i}|\nu_\alpha\rangle, \end{aligned} \tag{2.1}$$

where the subscript α runs over the flavor indices and i runs over the mass indices (1, 2, 3). The unitary matrix $U_{\alpha i}$ is known as the Pontecorvo-Maki-Nakagawa-Sakata matrix [15] and provides the mixing between the two basis. Explicitly:

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}, \tag{2.2}$$

where each entry is proportional to an angle $\theta_{\alpha i}$ and represents how much of a state $|\nu_\alpha\rangle$ enters in the composition of the state $|\nu_i\rangle$, and vice versa.

[1] The flavor of a neutrino is determined by the charged lepton involved in the corresponding weak reaction. For example, in a β decay are produced an electron and a neutrino (in addition to a proton), so the emitted particle is defined as an electron neutrino.

As a consequence of this mixing, a particle created with the flavor α has a certain probability, called Conversion Probability $P_{\alpha\beta}$, to be detected with the flavor β, where

$$P_{\alpha\beta} = \left|\langle\nu_\beta|e^{i\mathbf{p}L}|\nu_\alpha\rangle\right|^2 = \left|U^*_{\alpha i}e^{ip_iL}U_{\beta i}\right|^2. \tag{2.3}$$

Here, the second step is obtained by using Eqn.(2.1) two times on $|\nu_\alpha\rangle$ and considering the states normalized. The exponential $e^{i\mathbf{p}L}$ is the propagation term, assuming a stationary state for the beam, where $\mathbf{p}$ represents the momentum vector in the mass basis and L the distance traveled, so that $e^{i\mathbf{p}L}|\nu_i\rangle = e^{ip_iL}|\nu_i\rangle$. It is possible to show the conversion probability in a more explicit way by considering the Euler identity $e^{ix} = \cos x + i\sin x = 1 - 2\sin^2(\frac{x}{2}) + i\sin x$ and substituting $x = (p_i - p_k)L$, where p_k comes from expanding the squared modulus

$$\left|U^*_{\alpha i}e^{ip_iL}U_{\beta i}\right|^2 = \left(U^*_{\alpha i}e^{ip_iL}U_{\beta i}\right)\left(U^*_{\alpha k}e^{ip_kL}U_{\beta k}\right). \tag{2.4}$$

Then, using the unitarity of U, the properties of complex numbers and defining the phase $\varphi_i \equiv -\frac{1}{2}p_iL$, the Eqn (2.3) becomes:

$$\begin{aligned} P_{\alpha\beta} = \delta_{\alpha\beta} &- 4\sum_{i<k}\Re(U^*_{\alpha i}U_{\beta i}U_{\alpha k}U^*_{\beta k})\sin^2(\varphi_i - \varphi_k) \\ &+ 2\sum_{i<k}\Im(U^*_{\alpha i}U_{\beta i}U_{\alpha k}U^*_{\beta k})\sin(2(\varphi_i - \varphi_k)). \end{aligned} \tag{2.5}$$

Describing the conversion probability like this, it covers all possible cases and encloses the neutrino oscillation phenomena in itself. Analyzing it term by term, the Kronecker delta $\delta_{\alpha\beta}$ describes the survival of the particle flavor in the case of $\alpha = \beta$, while the second and the third term give to the phenomenon a dependence on the distance L and on the beam energy E through the phase φ_i. In particular, when the last term is not vanishing, it describes different oscillation probabilities for neutrinos and antineutrinos violating the CP symmetry: $P_{\alpha\beta} \neq P_{\bar\alpha\bar\beta}$.

Usually, however, the conversion probability is expressed in a more convenient way to make its variables more evident. Notice, in fact, that the phases difference can be rewritten, remembering that $E^2 = m_i^2 + p_i^2$ and defining the mass difference $\Delta m^2_{ik} \equiv m_i^2 - m_k^2$, as:

$$\begin{aligned} \varphi_i - \varphi_k &= -\frac{1}{2}(p_i - p_k)L = -\frac{1}{2}\frac{p_i^2 - p_k^2}{p_i + p_k}L = +\frac{1}{2}\frac{\Delta m^2_{ik}}{p_i + p_k}L \\ &= \frac{\Delta m^2_{ik}}{4E}L\left(1 + O\left(\frac{m_i^2}{E^2}\right)\right). \end{aligned} \tag{2.6}$$

Here, the ratio $\frac{m_i^2}{E^2}$ compares masses smaller than 1 eV with the energy of the beam, usually ranging between the MeV and the GeV order of magnitude, motivating the approximation at the first order. The survival probability becomes now:

$$P_{\alpha\alpha} = 1 - 4\sum_{i<k}|U_{\alpha i}|^2|U_{\alpha k}|^2\sin^2\left(\frac{\Delta m^2_{ik}L}{4E}\right), \tag{2.7}$$

where the CP violating term has been canceled because $\Im\left(|U_{\alpha i}|^2|U_{\alpha k}|^2\right) = 0$, telling that $P_{\alpha\alpha} = P_{\bar\alpha\bar\alpha}$ and disappearance experiments are not sensitive to CP violation.

Examining the simpler (but not real) case with only two lepton flavors, e and μ, Eqn. (2.7) can be revisited taking into account the form of the mixing matrix, which now defines a rotation in a two dimensional plane by an angle θ, defined the mixing angle:

$$U = \begin{pmatrix} U_{e1} & U_{e2} \\ U_{\mu 1} & U_{\mu 2} \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}. \tag{2.8}$$

Remembering that $\sin(2\theta) = 2\sin\theta\cos\theta$, then

$$P_{\alpha\alpha} = 1 - 4|U_{\alpha 1}|^2|U_{\alpha 2}|^2 \sin^2\left(\frac{\Delta m^2_{12} L}{4E}\right) = 1 - \sin^2(2\theta)\sin^2\left(\frac{\Delta m^2_{12} L}{4E}\right), \tag{2.9}$$

where $\sin^2(2\theta)$ controls the amplitude of oscillation while $\sin^2\left(\frac{\Delta m^2_{12} L}{4E}\right)$ is the term that oscillates as a function of the energy and the distance traveled, both parameters fixed by the experiment. Knowing that $P_{\alpha\beta} = 1 - P_{\alpha\alpha}$, let us observe that it is invariant under a global phase redefinition of the mixing matrix:

$$U_{\alpha k} \to e^{i\vartheta_\alpha} U_{\alpha k} e^{i\vartheta'_k}, \tag{2.10}$$

highlighting the fact that not all phases embedded in the unitary matrix are physical observables. While in the two flavor case this is a trivial result, since only one angle is present, in the three flavor case the mixing matrix accommodates three mixing angles and six phases, five of which can be reabsorbed with the phase redefinition, and thus are not physical.

It is important to note that this discussion is only valid for Dirac fermions, while describing neutrinos as Majorana fermions add two more physical phases to the counting due to a different definition of the mass term. Indeed, the Majorana mass term appearing in the Lagrangian is

$$\mathcal{L}_m = -\frac{1}{2} m_M (\nu\nu + \nu^c\nu^c) + h.c.\,, \tag{2.11}$$

where the two-components spinor notation is used, following the Ref. [16] conventions. The fields ν and ν^c are the left-handed neutrino field and its charge conjugation, respectively, while m_M is the Majorana mass of the particle. It is then easy to see that the global phase transformation

$$U(1): \quad \begin{array}{c} \nu \to e^{i\alpha}\nu \\ \nu^c \to e^{-i\alpha}\nu^c\,, \end{array} \tag{2.12}$$

gives the mass term a phase $e^{i2\alpha}$ instead of leaving it invariant. Therefore, in this case the three phases relative each to one neutrino flavor cannot be absorbed and the only unphysical phases are the one that can be absorbed by the charged leptons, whose Dirac mass terms are invariant under $U(1)$ transformation. Hence, the complete 3x3 mixing matrix U can be written explicitly as:

$$U = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{U_{23}} \underbrace{\begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix}}_{U_{13}} \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{U_{12}} \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\frac{\alpha}{2}} & 0 \\ 0 & 0 & e^{i\frac{\beta}{2}} \end{pmatrix}}_{P_M}, \tag{2.13}$$

where $c_{ij} = \cos(\theta_{ij})$, $s_{ij} = \sin(\theta_{ij})$ and the sub-matrices U_{ij} represent the rotation on the (i, j)-plane of the mass space. The three phases δ, α and β are, respectively, the CP violating phases and the two Majorana phases. From data measured during experiments such as Super-Kamiokande and KamLAND [6, 17] it has been possible to define numerical values for the PMNS matrix, which approximately reads:

$$U \approx \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}, \tag{2.14}$$

where it is interesting to note how the third column accommodates two entries describing the maximal mixing of ν_3 with the μ and τ flavors, meaning that, since the $\nu_\mu \leftrightarrow \nu_\tau$ oscillation amplitude is controlled by the term $\sin^2(2\theta_{23}) = 1$[2] , after a certain distance the whole beam has been converted from ν_μ to ν_τ.

However, those Majorana phases cancel out when computing the oscillation probability $P_{\alpha\beta}$, giving no information on the Dirac or Majorana nature of neutrinos and moreover, regarding the parameter counting, those experiments are sensitive to only one phase in either case.

Concerning the sensitivity on the masses, an important detail must be noted. In fact, survival and conversion probability are not sensitive directly to neutrino masses, but to their difference Δm^2_{ik} , making one of them dependent on the other two. Despite this lack of independence impedes a precise mass measurement, a hierarchy can be defined:

$$|\Delta m^2_{12}| < |\Delta m^2_{13}|, |\Delta m^2_{32}|, \tag{2.15}$$

giving rise to two possible scenarios as shown in Fig 2.1, assuming $m_2 > m_1$, without loss of generality.

2.2 The Seesaw Mechanism

As discussed before, the main problem that suggests New Physics in the neutrino sector concerns the neutrino mass, which is constrained to be $\frac{m_\nu}{m_e} < 10^{-6}$ [18] from the experimental bounds. Within the Standard Model, generating such extremely light masses via the standard Higgs mechanism would require unnaturally small Yukawa couplings ($y_\nu \sim 10^{-12}$), introducing a severe fine-tuning problem. Besides, questions arise when comparing the mixing behaviors of leptons and quarks. In fact, while the lepton mixing matrix U_{PMNS} exhibits two large angles close to the maximum mixing, the matrix governing the flavor mixing in the quark sector, the Cabibbo-Kobayashi-Maskawa matrix, or V_{CKM}, presents a strongly hierarchical and almost diagonal structure, implying quark transitions predominantly within the same generation:

$$V_{\text{CKM}} = \begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} \approx \begin{pmatrix} 0.974 & 0.225 & 0.004 \\ 0.225 & 0.973 & 0.042 \\ 0.009 & 0.041 & 0.999 \end{pmatrix}. \tag{2.16}$$

This stark contrast in the matrices structures suggests that the mechanism generating particle masses might be fundamentally different between the two

[2] This is because the angle $\theta_{23} = 45°$, obtained from the numerical values of U.

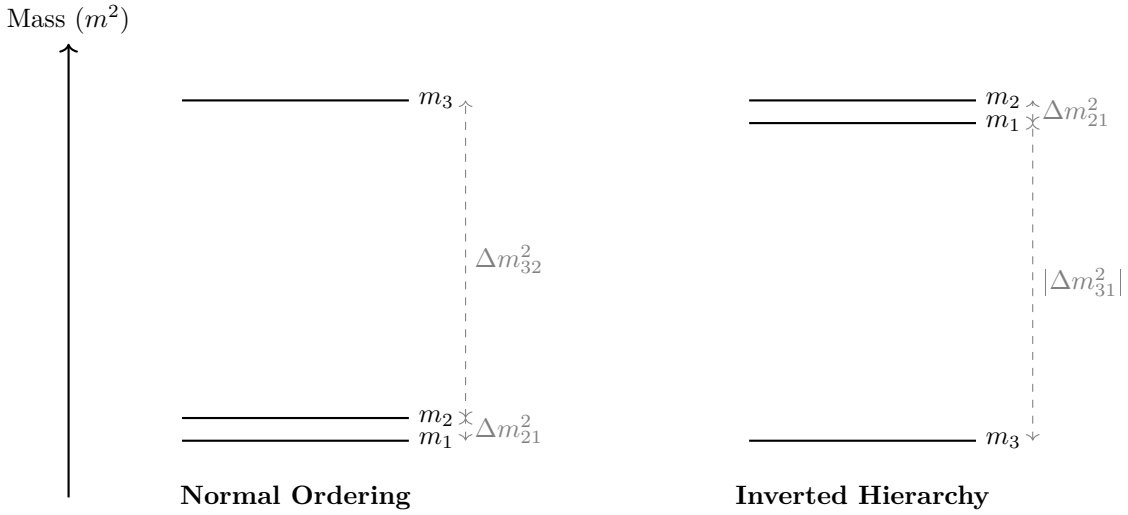


Figure 2.1: Neutrino mass spectrum: Normal Ordering (NO) vs Inverted Hierarchy (IH). The diagram illustrates the two possible arrangements of the neutrino mass eigenstates. In the Normal Ordering scenario (left), $m_1 < m_2 < m_3$, which reflects the standard mass hierarchy observed in the charged lepton and quark sectors. In the Inverted Hierarchy scenario (right), $m_3 < m_1 < m_2$, meaning the two closely spaced states m_1 and m_2 are heavier than the third state. In both cases, the mass splitting Δm^2_{21} is experimentally known to be strictly positive and significantly smaller than the absolute value of $|\Delta m^2_{31}|$

sectors and hints at the presence of new flavor symmetries constraining lepton mixing [19]. The Seesaw mechanism provides an elegant and natural solution explaining the suppression of the neutrino mass scale through the introduction of heavy energy scales.

The reason why the SM predicts vanishing neutrino masses, causing the discussed discrepancy with observations, comes directly from the Yukawa interaction Lagrangian. To understand SM predictions and how the seesaw model can extend it with New Physics potentially solving the problem, it is needed to review the SM particle content and the Higgs mechanism to give them mass. The SM Lagrangian, describing particles properties, masses and interactions, originates from the symmetry gauge group $G_{SM} = SU(3)_c \times SU(2)_L \times U(1)_Y$, where the three sub-groups represent, respectively, the color group, which acts on quarks and describe the strong force, the weak (or Left handed) group, which acts only on left handed fermions, and the Hypercharge group, which acts on all the fermions and combines with $SU(2)_L$ producing the observed electric force. Hence, every particle can be described uniquely by their transformation under those groups separately, for example:

$$l = \begin{pmatrix} \nu \\ e \end{pmatrix} \sim \left(1, 2, -\frac{1}{2}\right). \tag{2.17}$$

This compact notation tells that the so called lepton doublet, composed by a left handed neutrino ν and a left handed charged fermion e, transforms as a singlet under $SU(3)_c$, and so is invariant under its actions, while under the action of $SU(2)_L$ it transforms in the 2 representation (so as a doublet), meaning that the generators of the symmetry are $T_i = \frac{1}{2}\sigma_i$, where σ_i are the Pauli matrices. The third quantum number correspond to the hypercharge Y acquired under the $U(1)_Y$ action, assigned to the lepton so that it can recover the observed electric charge Q through the relation

$$Q = T_3 + Y. \tag{2.18}$$

In this case, the electric charge of the doublet (and so, of its components) are, explicitly:

$$Q = \frac{1}{2}\sigma_3 + Y = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \end{pmatrix} + \begin{pmatrix} -\frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix}, \tag{2.19}$$

corresponding to the expected values for the neutrino, which is neutral, and the electron, for which $q(e) = -1$ in natural units.

Focusing on the lepton sector, the standard particle content consists only of the lepton doublet l_α already seen and the charged lepton singlet

$$e^c_\alpha \sim (1, 1, 1), \tag{2.20}$$

corresponding to the charge-conjugate of the right handed electron. By using the two-component spinor formalism and defining all fields as left-handed Weyl spinors, e^c_α is described as a singlet under $SU(2)_L$, carrying a positive unit hypercharge. It is important to notice that, unlike the quark sector, the lepton sector does not possess singlets for both "up-type" and "down-type", lacking a neutral singlet partner.

Here it comes the assumption of the Type-I Seesaw mechanism, proposing the extension of the lepton particle spectrum with the introduction of a right handed neutrino for each generation, also known as sterile neutrino or Heavy Neutral Lepton (HNL), represented in the Lagrangian by left-handed conjugate spinor N_α (often denoted as ν_α^c in the literature). This field must be invariant under both $SU(3)_c$ and $SU(2)_L$, like the charged lepton singlet, but in order to be neutral it must have vanishing hypercharge:

$$N_\alpha \sim (1, 1, 0)\,. \tag{2.21}$$

The new particle is then built to transform trivially under the whole G_{SM} symmetry group, preventing it from coupling with any gauge boson and making it completely sterile from the SM gauge interaction point of view, but allowing the introduction of a gauge invariant bare mass term for the neutrino field. The direct coupling of the HNL with the lepton doublet, however, would not preserve the gauge invariance of the Lagrangian if it's not mediated by a scalar field able to absorb the phases produced during their gauge transformation. This happens to be true for every fermion of the SM and the needed scalar is the Higgs field, defined as

$$\varphi \sim (1, 2, +\frac{1}{2}). \tag{2.22}$$

The interaction term between the two lepton fields and the Higgs, called Yukawa interaction, then reads:

$$\mathcal{L}_{Yuk} = -y_\nu^{\alpha\beta}\varepsilon_{ab} l_\alpha^a \varphi^b N_\beta + h.c., \tag{2.23}$$

where y_ν is the Yukawa coupling and ε_{ab} the antisymmetric tensor acting on $SU(2)_L$ and $a, b = 1, 2$ are its indices. The indices α, β define the flavor eigenstate. All the components of this term transform trivially under $SU(3)_c$, while the $SU(2)_L$ group acts like:

$$l^a \to U^a{}_c l^c, \qquad \varphi^b \to U^b{}_d \varphi^d, \tag{2.24}$$

from which it follows that

$$\varepsilon_{ab} l^a \varphi^b \to \varepsilon_{ab}(U^a{}_c l^c)(U^b{}_d \varphi^d) = (\varepsilon_{ab} U^a{}_c U^b{}_d) l^c \varphi^d = (\det U)\varepsilon_{cd} l^c \varphi^d, \tag{2.25}$$

where $\det(U) = 1$, leaving the term unchanged. Finally, the group $U(1)_Y$ acts giving to each field a phase $l \to e^{i\alpha Y_l} l$, thus the term will acquire a total phase equal to the sum of the individual ones:

$$Y(l) + Y(\varphi) + Y(N) = 0, \tag{2.26}$$

concluding that the Yukawa interaction term is gauge invariant before the electroweak symmetry breaking.

However, since the N field is free from any gauge charge, the Yukawa interaction is not the only gauge invariant term we can add to the Lagrangian. To be completely general, indeed, also the Majorana mass term must be considered:

$$\mathcal{L}_M = -\frac{1}{2} M^{\alpha\beta} N_\alpha N_\beta + \text{h.c.}, \tag{2.27}$$

where $M^{\alpha\beta}$ is the Majorana mass matrix, symmetric in the flavor indices due to the anticommutation rule of the fermion fields $N_\alpha N_\beta = N_\beta N_\alpha$. Unlike Dirac

masses, which require both a particle and its antiparticle to be defined, Majorana masses circumvent the requirement by considering N its own antiparticle.

This clearly rises a problem in the conservation of the lepton number. In fact, the Yukawa term satisfies one global symmetry not shown yet, $U(1)_L$, once the fields l and N are associated with a lepton number $L = +1$ and $L = -1$ respectively. Consequently, the violation of this symmetry group arises naturally from the Majorana mass term, by two units ($\Delta L = 2$), allowing in this way the conversion process of a neutrino into an antineutrino. This violation leads to profound consequences, such as the possibility for a neutrinoless double β- decay or a possible explanation for leptogenesis.

From the Yukawa interaction with the Higgs boson, fermions can obtain a Dirac mass term once the theory undergoes the electroweak symmetry breaking process and the Higgs scalar gains a vacuum expectation value (VeV): $\langle\varphi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\v\end{pmatrix}$. The Yukawa term becomes

$$\mathcal{L}_{Yuk} = -\frac{y_\nu^{\alpha\beta}}{\sqrt{2}}\begin{pmatrix}\nu_\alpha\\e_\alpha\end{pmatrix}\begin{pmatrix}0&1\\-1&0\end{pmatrix}\begin{pmatrix}0\\v\end{pmatrix}N_\beta + h.c. = -\frac{y_\nu^{\alpha\beta}v}{\sqrt{2}}\,\nu_\alpha N_\beta + h.c., \quad (2.28)$$

which is now a quadratic term in the neutrino fields, and representing thus a mass term, where the Dirac mass is defined as

$$(m_D)_{\alpha\beta} = \frac{y_\nu^{\alpha\beta}v}{\sqrt{2}}. \quad (2.29)$$

Now, the complete Lagrangian for the neutrino sector reads:

$$\mathcal{L}_{\text{mass}} = -(m_D)_{\alpha\beta}\,\nu_\alpha N_\beta - \frac{1}{2}M^{\alpha\beta}N_\alpha N_\beta + \text{h.c.}. \quad (2.30)$$

A more convenient and compact writing requires the introduction of the spinor vector $\mathbf{\Psi} = \begin{pmatrix}\nu\\N\end{pmatrix}$, collecting the three mass states of both ν and N^c fields, allowing to summarize the two mass matrix as a single block matrix.

$$\mathcal{L}_{\text{mass}} = -\frac{1}{2}\mathbf{\Psi}^T\mathcal{M}\mathbf{\Psi} + \text{h.c.}, \quad (2.31)$$

where

$$\mathcal{M} = \begin{pmatrix}0&m_D\\m_D^T&M\end{pmatrix} \quad \text{and} \quad m_D = \begin{pmatrix}(m_D)_{e1}&(m_D)_{e2}&(m_D)_{e3}\\(m_D)_{\mu1}&(m_D)_{\mu2}&(m_D)_{\mu3}\\(m_D)_{\tau1}&(m_D)_{\tau2}&(m_D)_{\tau3}\end{pmatrix}. \quad (2.32)$$

The total mass matrix $\mathcal{M}$ is a 6x6 complex and symmetric matrix and contains two energy scales. The Seesaw mechanism assumes that while the Dirac mass is constrained by the electroweak scale, i.e. $m_D \lesssim 10^2$ GeV, the Majorana mass scale can be arbitrarily large, not confined by any symmetry. So, without loss of generality, we can assume $M \gg m_D$ and define thus a small perturbative parameter $\epsilon \sim m_D M^{-1} \ll 1$, allowing to perturbatively block-diagonalize the mass matrix

$$\mathcal{M}_{\text{diag}} \approx \begin{pmatrix}m_{\text{light}}&0\\0&M_{\text{heavy}}\end{pmatrix}, \quad (2.33)$$

where the two mass matrices on the diagonal blocks are defined as:

$$\begin{aligned} m_{\text{light}} &\simeq -m_D M^{-1} m_D^T, \\ M_{\text{heavy}} &\simeq M. \end{aligned} \tag{2.34}$$

Explicit computation is shown in the next section, in the case of only two sterile neutrinos.

This result explains, as expected, the origin of a such light but not vanishing mass for the active neutrinos. In fact, despite the Majorana mass results almost unconstrained, assuming $M \sim$ GeV and very small Yukawa couplings would allow to reproduce correctly the phenomenology while keeping the right-handed neutrinos within an experimentally accessible scale.

The type-I Seesaw Lagrangian changes deeply the accidental symmetries of the theory, due to the presence of the right handed singlets Majorana mass term together with their Yukawa interaction. For the active sector, the SM preserves four accidental[3] global symmetries, namely the conservation of the baryon number B, corresponding to the number of quarks in the system divided by three, and the three lepton numbers L_e, L_μ and L_τ, separated by flavors. The sum of the flavor lepton numbers is known as the total lepton number and its difference with the baryon number is still conserved by the global symmetry $U(1)_{B-L}$. By adding the sterile neutrino N, these symmetries can still be conserved within the Dirac sector, since its Yukawa term conserves the total lepton number but, as previously shown, the Majorana mass term introduces a $\Delta L = 2$ difference, breaking explicitly the lepton number conservation and, consequentially, the $U(1)_{B-L}$ symmetry, opening the door to the leptogenesis mechanism[20].

A second important violation brought by the seesaw model concerns the CP symmetry, a fundamental symmetry coming out from the application of two discrete transformations: charge conjugation, which swaps particles with their antiparticle partners changing thus the sign of all conserved charges, and parity, which inverts the spatial coordinates of the system, and particles' momentum and chirality with them. Preserving the CP symmetry means that the theory laws are perfectly symmetric for particles and antiparticles. To understand how CP gets violated in the neutrino sector, a parameter counting for the Yukawa matrix is needed.

Considering the Majorana mass term $\mathcal{L}_M = -\frac{1}{2}(N)^T M N + \text{h.c.}$, a unitary transformation on the fields

$$N \to UN \tag{2.35}$$

is always possible, allowing the mass matrix diagonalisation $M_{\text{diag}} = U^T M U$, with the positive and real eigenvalues M_1, M_2, M_3 corresponding to the sterile masses. At this point, the Yukawa interaction is left with all the complexity on itself. In particular, the 3x3 complex Yukawa matrix,

$$y_\nu^{\alpha\beta} = \begin{pmatrix} |y_{ee}|e^{i\phi_{ee}} & |y_{e\mu}|e^{i\phi_{e\mu}} & |y_{e\tau}|e^{i\phi_{e\tau}} \\ |y_{\mu e}|e^{i\phi_{\mu\mu}} & |y_{\mu\mu}|e^{i\phi_{\mu\mu}} & |y_{\mu\tau}|e^{i\phi_{\mu\tau}} \\ |y_{\tau e}|e^{i\phi_{\tau e}} & |y_{\tau\mu}|e^{i\phi_{\tau\mu}} & |y_{\tau\tau}|e^{i\phi_{\tau\tau}} \end{pmatrix}, \tag{2.36}$$

[3]This term is used for symmetries which emerge a posteriori from the theory instead of being imposed a priori for the formulation of the theory. They appear when no term in the Lagrangian can break them without violating a gauge symmetry at the same time.

contains nine real parameters $|y_{\alpha\beta}|$ and nine complex phases $e^{i\phi_{\alpha\beta}}$.

Unlike for the U_{PMNS} matrix in the Dirac neutrino case, which gets five phases out of six absorbed by a redefinition of the leptonic fields, here the HNLs phases are constrained by the Majorana mass term, making impossible to absorb them without breaking the diagonality of the mass matrix. The only fields free to be redefined are the right handed charged leptons e^c_α, which allow to get rid of only three phases, one per flavor. The Yukawa matrix is then left with six physical phases responsible for CP violation. Three of these phases manifest in the U_{PMNS} matrix in the case of Majorana neutrinos, while the remaining three control extremely high energy processes such as in the primordial Universe and the ones responsible for leptogenesis. Therefore, the Seesaw model is able to provide a source not only for the neutrino masses, but also for the baryon asymmetry and sterile neutrinos could even play the role of dark matter candidates, answering multiple questions simultaneously, if considering a model with three copies of N, known as the Neutrino Minimal Standard Model (νMSM)[7].

Even though it is natural to assume one copy of sterile neutrino for each lepton generation, there is nothing imposing all three generations to have a non vanishing mass, from a phenomenological point of view. Oscillation experiments, as a matter of fact, are sensitive to only two mass differences, requiring thus *at least* two non vanishing neutrino masses. It is then possible to avoid introducing a third right handed neutrino without losing any generality. This assumption is the starting point for the Minimal Seesaw Model (2RHN model)[14], where only two neutrino singlets are introduced, reducing the free parameters but leaving the equations unchanged. A detailed computation is shown in the next section.

2.3 Sterile Neutrino Lagrangian in the Minimal Seesaw Model

As said before, sterile neutrinos can contribute to active neutrino mass by mixing with them in the Yukawa interaction term, and they have their own mass (see Eqn(2.30)). Such mass plays an important role when it comes to detecting them in the experiments, but theory leaves it almost unconstrained, giving no precise predictions inside the $m_\nu \sim 0.1$ eV $\lesssim M_N \lesssim 10^{15}$ GeV range. In the following, the notation of [10] will be used for fermion fields and mass eigenstate fields. For the following discussion we will be interested in the mass range 0.1 GeV $\lesssim M_N \lesssim 10$ GeV, at which scale mixing parameters are expected to be small.

Following the Minimal Seesaw Model, the Lagrangian for the New Physics reads:

$$\begin{aligned}\mathcal{L}_N = {} & iN_1'^{\dagger}\bar{\sigma}^\mu\partial_\mu N_1' + iN_2'^{\dagger}\bar{\sigma}^\mu\partial_\mu N_2' - \left(MN_1'N_2' + \frac{\mu_1}{2}N_1'N_1' + \frac{\mu_2}{2}N_2'N_2' + \text{h.c.}\right) \\ & - \left[\tilde{\varphi}^\dagger l'(y_N^1 N_1' + y_N^2 N_2') + \text{h.c.}\right]\end{aligned} \tag{2.37}$$

where the primed notation refers to the fermion fields in the flavor basis, written in the two-component spinor notation using Ref. [16] conventions. In the first line of the equation we present the kinetic and the mass terms for the two RH neutrinos, where $\mu_{1,2}, M$ are the Majorana masses, while in the second part of the Lagrangian their interactions are shown through the Yukawa portal, where

only one lepton flavor is considered for the moment.

An analysis of the Lagrangian symmetries is required to make considerations on sterile neutrino masses. In this context it is easy to show that the kinetic term is invariant under a $SU(2)_N \times U(1)_N$ transformation, once the doublet $\mathbf{N'} \equiv \begin{pmatrix} N_1' \\ N_2' \end{pmatrix}$ is defined. The transformations are defined as follows:

$$\begin{aligned} SU(2)_N: &\quad \mathbf{N'}_a \to (V_N)_a^b \mathbf{N'}_b, \\ U(1)_N: &\quad \mathbf{N'} \to e^{i\alpha_N} \mathbf{N'}, \end{aligned} \tag{2.38}$$

therefore:

$$\mathcal{L}_k = i\mathbf{N'}^\dagger \sigma^\mu \partial_\mu \mathbf{N'} \to i\mathbf{N'}^\dagger V_N^\dagger e^{-i\alpha_N} \sigma^\mu \partial_\mu V_N e^{i\alpha_N} \mathbf{N'}, \tag{2.39}$$

which corresponds to the initial kinetic term since $V_N^\dagger V_N = \mathbb{1}$. The mass term, on the other hand, breaks both groups, but it is possible to preserve a subgroup of $SU(2)_N$ in some special cases. In order to write the term using the $\mathbf{N'}$ doublet notation, we can define the mass matrix as

$$\mathfrak{M} = \begin{pmatrix} \mu_1 & M \\ M & \mu_2 \end{pmatrix} = \begin{pmatrix} -m_1 + im_2 & m_3 \\ m_3 & m_1 + im_2 \end{pmatrix}, \tag{2.40}$$

where the second step corresponds to a possible parametrization of such matrix and it is possible to write it in a more compact way as $\mathfrak{M}^{ab} = m_A(\sigma_A \varepsilon)^{ab}$. The mass term reads now:

$$\mathcal{L}_m = \frac{1}{2} m_A \mathbf{N}_a'^T (\sigma_A \varepsilon)^{ab} \mathbf{N}_b'. \tag{2.41}$$

Let us observe that the mass matrix corresponds to a three dimensional vector in the $SU(2)_N$ space when written in this way, and its components are the masses m_A. This representation becomes particularly useful in the case of real masses, since it is always possible to align the vector along one axes without loss of generality, making it invariant under a rotation in the plane of the remaining two axis. A subgroup $U(1)'_N$ is then recovered, acting as

$$\mathbf{N}' \to e^{\frac{i}{2}\sigma_3\theta_3} \mathbf{N}' = \begin{pmatrix} e^{\frac{i}{2}\theta_3} \mathbf{N}_1' \\ e^{-\frac{i}{2}\theta_3} \mathbf{N}_2' \end{pmatrix}, \tag{2.42}$$

where the real-triplet $\mathfrak{M}^{ab}$ in the third component direction, $m1 = m2 = 0, m3 = M$ has been considered, explaining the presence of the Pauli matrix σ_3 in the transformation and giving the sterile neutrinos a conserved charge $q(\mathbf{N}'_{1,2}) = \pm 1$ under the symmetry group. The charges come from the diagonal entries of σ_3.

It is interesting to see how the mass matrix can be diagonalized in each case. For real masses, it is easy to show that

$$\mathfrak{M}^2_{diag} = \mathfrak{M}\mathfrak{M}^\dagger = \begin{pmatrix} M_1^2 & 0 \\ 0 & M_2^2 \end{pmatrix}, \tag{2.43}$$

where $M_1^2 = M_2^2 = m_1^2 + m_2^2 + m_3^2$.

In the general case of complex m_A the $SU(2)_N$ symmetry group gets fully broken and the mass eigenvalues $M_1 \neq M_2$. Computation of $\mathcal{M}_{diag}$ can be

done easily by considering $m_A = |m_A|e^{i\alpha_A}$ explicitly and the results appear in Section 2 of Ref.[10].

So, every term of the Lagrangian studied up to now is invariant under $U(1)'_N$, when asking for $\mu_{1,2} = 0$. The last one left to consider is the Yukawa interaction term. To ensures its invariance under $U(1)'_N$ without putting the couplings $y_N^{1,2} \to 0$, the lepton doublet l' must transform with a charge ± 1 able to absorb the ones generated by the HNLs. Since the two neutrinos transform with different charges, the symmetry group invariance is recovered only in the case of one vanishing coupling, which will be freely chosen to be y_n^1 so that we can assign $q(l') = +1$.

With these new conditions the $U(1)'_N$ group acts effectively as a lepton number symmetry and it imposes a vanishing mass for the active neutrino ν' together with a non zero Dirac mass for the sterile field. A vanishing active neutrino mass $m_\nu = 0$ arises from observing that a Majorana mass term $\frac{1}{2}m_\nu \nu'\nu'$ would acquire a phase of $e^{+2i\alpha}$ after the lepton number symmetry transformation, requiring to exclude it to maintain the invariance of the Lagrangian. Another explanation for this requires the definition of the mass matrix of the system:

$$\mathcal{M} = \begin{pmatrix} 0 & y_N^1 v & y_N^2 v \\ y_N^1 v & \mu_1 & M \\ y_N^2 v & M & \mu_2 \end{pmatrix} = \begin{pmatrix} 0 & 0 & y_N^2 v \\ 0 & 0 & M \\ y_N^2 v & M & 0 \end{pmatrix}, \tag{2.44}$$

whose eigenvalues are $\lambda_1 = 0$ and $\lambda_{1,2} = \pm\sqrt{M^2 + (y_N^2 v)^2}$. It is now easy to see that requiring $\lambda_1 \neq 0$, i.e. obtaining a small Majorana mass m_ν, corresponds again to breaking the $U(1)_N$ symmetry.

Having now details on the symmetry breaking pattern of the model gives some insight on the parameters relative scale. In fact, by the 't Hooft technical naturalness principle, if the system recovers a new symmetry when some parameter vanishes, then that parameter is expected to remain small (with respect to the theory energy scale), without the quantum corrections to make it explode[21]. In this case, therefore, we expect the sterile mass scale M to be as small as desired, protected by the $SU(2)_N$ symmetry recovered for $M \to 0$. In addition, also $\Delta M = M_2 - M_1$ is required to be even smaller due to the recovery of $U(1)_N$ symmetry when it vanishes.

2.3.1 Mass matrix diagonalisation and mixing angles

Consider the complete and symmetric 3x3 mass matrix written in Eqn.(2.44), which can be expressed in a more compact way as:

$$\mathcal{M} = \left(\begin{array}{c|c} 0 & m_D^T \\ \hline m_D & \mathfrak{M} \end{array}\right), \tag{2.45}$$

where $m_{Di} = y_N^i v$. It can be diagonalised with a unitary matrix U through the relation $U^T \mathcal{M} U = \mathcal{M}_{diag}$, where the entries of the diagonal matrix correspond to the mass eigenvalues of ν, N_1, N_2, This is the matrix that will appear in the mass term of the physical Lagrangian. Hence, using the diagonalisation relation, it is possible to relate the physical fields with the primed ones:

$$\mathcal{L}_{mass}^{phys} = \chi^T \mathcal{M}_{diag} \chi = \chi^T U^T \mathcal{M} U \chi = \chi'^T \mathcal{M} \chi', \tag{2.46}$$

where χ and χ' are some field in the mass eigenvalues basis and in the flavor one, respectively. In the sterile neutrino context, they can be defined as

$$\chi = \begin{pmatrix} \nu \\ N_1 \\ N_2 \end{pmatrix} \quad \text{and} \quad \chi' = \begin{pmatrix} \nu' \\ N_1' \\ N_2' \end{pmatrix}, \tag{2.47}$$

which are then related by $\chi' = U\chi$. It is clear, thus, that the matrix U is essential to find the physical fields, corresponding to the detected particles.

Start by defining a general block form of the matrix

$$U = \left(\begin{array}{c|c} a & \boldsymbol{\theta}^\dagger \\ \hline \mathbf{b} & \mathfrak{U} \end{array}\right), \tag{2.48}$$

considering a a complex number, $\mathbf{b}$ and $\boldsymbol{\theta}$ are 2-component complex vectors and $\mathfrak{U}$ is a general 2×2 matrix, and use the unitary property to solve the equation $U^\dagger U = 1$ to find a and $\mathbf{b}$. Solutions will give:

$$\begin{cases} a = \sqrt{1 - \boldsymbol{\theta}^\dagger \boldsymbol{\theta}} e^{i\alpha} \\ \mathbf{b} = -\frac{\mathfrak{U}\boldsymbol{\theta}}{\sqrt{1-\boldsymbol{\theta}^\dagger\boldsymbol{\theta}}} e^{i\alpha} \\ \mathfrak{U}^\dagger\mathfrak{U} = \mathbb{1}_{2x2} - \boldsymbol{\theta}\boldsymbol{\theta}^\dagger \end{cases} \tag{2.49}$$

where α is the generic complex phase of a and the last condition is used to determine $\mathfrak{U}$. Since any hermitian matrix can be diagonalized via a unitary transformation, the matrix $\mathfrak{U}$ must satisfy:

$$\mathfrak{U}^\dagger\mathfrak{U} = V_\theta \begin{pmatrix} 1 - \boldsymbol{\theta}^\dagger\boldsymbol{\theta} & 0 \\ 0 & 1 \end{pmatrix} V_\theta^\dagger \quad \longrightarrow \quad \mathfrak{U} = V \begin{pmatrix} \sqrt{1 - \boldsymbol{\theta}^\dagger\boldsymbol{\theta}} & 0 \\ 0 & 1 \end{pmatrix} V_\theta^\dagger \tag{2.50}$$

where V_θ is the unitary rotation matrix with columns given by the eigenvectors of $\boldsymbol{\theta}\boldsymbol{\theta}^\dagger$ and V is an arbitrary unitary rotation that can be defined as

$$V = \begin{pmatrix} e^{i\beta} & 0 \\ 0 & e^{i\gamma} \end{pmatrix} \begin{pmatrix} \cos\theta_3 & -\sin\theta_3 \\ \sin\theta_3 & \cos\theta_3 \end{pmatrix} \begin{pmatrix} e^{i\delta} & 0 \\ 0 & e^{-i\delta} \end{pmatrix}, \tag{2.51}$$

where the phases β, γ, together with α can be absorbed by a redefinition of the fields ν', N_1', N_2' , leaving only $\boldsymbol{\theta}, \theta_3$ and δ as physical parameters.

Finally, using Eqn. (2.49) to replace a and $\mathbf{b}$ entries of the U matrix, it is possible to compute exactly the primed fields with respect to the physical ones:

$$\begin{pmatrix} \nu' \\ \mathbf{N}' \end{pmatrix} = \begin{pmatrix} \sqrt{1 - \boldsymbol{\theta}^\dagger\boldsymbol{\theta}} & \boldsymbol{\theta}^\dagger \\ -\frac{\mathfrak{U}\boldsymbol{\theta}}{\sqrt{1-\boldsymbol{\theta}^\dagger\boldsymbol{\theta}}} & \mathfrak{U} \end{pmatrix} \begin{pmatrix} \nu \\ \mathbf{N} \end{pmatrix}, \tag{2.52}$$

leading to,

$$\begin{aligned} \nu' &= \sqrt{1 - \boldsymbol{\theta}^\dagger\boldsymbol{\theta}}\, \nu + \boldsymbol{\theta}\, \boldsymbol{N}, \\ \mathbf{N}' &= \mathfrak{U}\mathbf{N} - \frac{\mathfrak{U}\boldsymbol{\theta}\, \nu}{\sqrt{1 - \boldsymbol{\theta}^\dagger\boldsymbol{\theta}}}. \end{aligned} \tag{2.53}$$

Finally, it is possible to block-diagonalize the mass matrix (2.44) in the perturbative parameter $\varepsilon = \mathfrak{M}^{-1} m_D$ as seen in Section 2.2. Since the mixing

angles are expected to be small, all quadratic terms in $\boldsymbol{\theta}$ will be neglected in the following, therefore the approximation $\sqrt{1-\boldsymbol{\theta}^\dagger\boldsymbol{\theta}} \simeq 1$ is taken. Starting from $U^T\mathcal{M}U = \mathcal{M}_{diag}$, the off-diagonal block to the left reads

$$\boldsymbol{\theta}^* m_D^T \mathfrak{U}\boldsymbol{\theta} + \mathfrak{U}^T m_D - \mathfrak{U}^T\mathfrak{M}\mathfrak{U}\boldsymbol{\theta} = \mathbf{0}, \tag{2.54}$$

where the first term is suppressed by two powers of $\boldsymbol{\theta}$, assumed to be small, and can therefore be neglected in a first order approximation. The remaining identity establishes the condition for the block-diagonalisation

$$\mathfrak{U}\boldsymbol{\theta} \simeq \mathfrak{M}^{-1} m_D. \tag{2.55}$$

Using this result, it is possible to define (at leading order) the active neutrino mass eigenvalue from the first diagonal block in the diagonalisation equation:

$$m_\nu = -m_D^T\mathfrak{U}\boldsymbol{\theta} - \boldsymbol{\theta}^T\mathfrak{U}^T m_D + \boldsymbol{\theta}^T\mathfrak{U}^T\mathfrak{M}\,\mathfrak{U}\boldsymbol{\theta} \simeq -m_D^T\mathfrak{M}^{-1}m_D, \tag{2.56}$$

which is suppressed at the second order in the perturbative parameter.

It is possible to solve exactly the unitary matrix U in the limit that recovers $U(1)_N$ symmetry group, giving a precise expression for the mixing angle vector $\boldsymbol{\theta}$ and the 2x2 matrix $\mathfrak{U}$. The mass matrix is then described by the second result of Eqn. (2.44) and its eigenvalues are the Dirac masses $m_\nu = 0, M_D = \sqrt{M^2 + m_{D2}^2}$. Realizing that the unitary matrix U can be built out of $\mathcal{M}_{diag}$ eigenvectors:

$$U = \begin{pmatrix} \mathbf{v}_0 & \mathbf{v}_1 & \mathbf{v}_2 \end{pmatrix} = \begin{pmatrix} c & \frac{1}{\sqrt{2}}s & -i\frac{1}{\sqrt{2}}s \\ -s & \frac{1}{\sqrt{2}}c & -i\frac{1}{\sqrt{2}}c \\ 0 & \frac{1}{\sqrt{2}} & i\frac{1}{\sqrt{2}} \end{pmatrix}, \tag{2.57}$$

where is has been used the short hand notation $c = \frac{M}{M_D}$, $s = \frac{m_{D2}}{M_D}$. The mixing angles are then:

$$\boldsymbol{\theta} = \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} is \\ s \end{pmatrix}, \tag{2.58}$$

and from the Eqn (2.53) the relation between physical and primed fields become

$$\begin{aligned} \nu' &= -\frac{i}{\sqrt{2}}s\,N_1 + \frac{1}{\sqrt{2}}s\,N_2 + c\,\nu, \\ N_1' &= -\frac{i}{\sqrt{2}}c\,N_1 + \frac{1}{\sqrt{2}}c\,N_2 - s\,\nu, \\ N_2' &= \frac{1}{\sqrt{2}}(i\,N_1 + N_2). \end{aligned} \tag{2.59}$$

Let us consider now a particular way of slightly breaking $U(1)_N$ by a small mass splitting parameter

$$\delta \equiv \frac{M_2 - M_1}{M_2 + M_1}, \tag{2.60}$$

such that

$$\mathcal{M} = \begin{pmatrix} 0 & 0 & m_{D2} \\ 0 & 0 & M \\ m_{D2} & M & 2\delta M_s \end{pmatrix}, \tag{2.61}$$

where $M_s \equiv \frac{M_D}{\sqrt{1-\delta^2}}$ and, therefore, $M_{diag} = diag(0, M_s(1-\delta), M_s(1+\delta))$. This violation is particularly convenient since the theory provides now two massive non-degenerate Majorana neutrinos which depend only on the mass splitting parameter, while the active neutrino mass remains vanishing. The eigenvectors matrix can be computed explicitly:

$$U = \begin{pmatrix} c & -\frac{i}{\sqrt{2}} s\sqrt{1+\delta} & \frac{1}{\sqrt{2}} s\sqrt{1-\delta} \\ -s & -\frac{i}{\sqrt{2}} c\sqrt{1+\delta} & \frac{1}{\sqrt{2}} c\sqrt{1-\delta} \\ 0 & \frac{i}{\sqrt{2}}\sqrt{1-\delta} & \frac{1}{\sqrt{2}}\sqrt{1+\delta} \end{pmatrix}, \tag{2.62}$$

and therefore we obtain

$$\boldsymbol{\theta} = \frac{1}{\sqrt{2}} \begin{pmatrix} i\, s\sqrt{1+\delta} \\ s\sqrt{1-\delta} \end{pmatrix}, \quad \mathfrak{U} = \frac{1}{\sqrt{2}} \begin{pmatrix} -i\, c\sqrt{1+\delta} & c\sqrt{1-\delta} \\ i\sqrt{1-\delta} & \sqrt{1+\delta} \end{pmatrix}. \tag{2.63}$$

2.3.2 The three lepton flavor case

Phenomenological observations of neutrino oscillations exhibit strong dependence on lepton flavor, which was neglected in the previous computation. To proceed with a more physical model, consider now the Lagrangian with all the three lepton flavor l_i, promoting in this way also the system mass to a 5×5 matrix so that it can accommodate all the possible combination between the flavors and the two HNLs. While its block structure remains the same as in Eqn. (2.45), the single term in the $(1,1)$ position representing the active neutrino mass m_ν is now a 3×3 matrix with all vanishing elements and the vector m_D becomes

$$m_D = \begin{pmatrix} 0 & 0 & 0 \\ m_{2e} & m_{2\mu} & m_{2\tau} \end{pmatrix}, \tag{2.64}$$

where the first row reflects the $U(1)_N$ conserving limit where $y_N^{1\alpha} = \mu_{1,2} = 0$ and the masses of the second row are defined, as in the single flavor case, as $m_{2\alpha} = y_N^{2\alpha}\, v$. The sterile mass matrix $\mathfrak{M}$ does not change and the diagonalization of the total mass matrix results in, as before, $\mathcal{M}_{diag} = (0, 0, 0, M_D, M_D)$, where

$$M_D = \sqrt{M^2 + m_{2e}^2 + m_{2\mu}^2 + m_{2\tau}^2}\,. \tag{2.65}$$

To allow such diagonalization, also the unitary matrix U is now 5×5, and consequently the mixing angle vector is promoted to a 2×3 matrix:

$$\boldsymbol{\theta} = \begin{pmatrix} \theta_{1e} & \theta_{1\mu} & \theta_{1\tau} \\ \theta_{2e} & \theta_{2\mu} & \theta_{2\tau} \end{pmatrix}. \tag{2.66}$$

The easiest way to determine the diagonalizing matrix U is divided in two steps, corresponding to diagonalizing the flavor space and the sterile space separately. First, define the mixing parameters as $s_i = \sin\theta_i$ and $c_i = \cos\theta_i$, where:

$$\begin{aligned} s_\tau &\equiv \frac{m_{2\tau}}{\sqrt{m_{2\tau}^2 + M^2}} && \text{and} \quad c_\tau \equiv \frac{M}{\sqrt{m_{2\tau}^2 + M^2}}, \\ s_\mu &\equiv \frac{m_{2\mu}}{\sqrt{m_{2\mu}^2 + m_{2\tau}^2 + M^2}} && \text{and} \quad c_\mu \equiv \frac{\sqrt{m_{2\tau}^2 + M^2}}{\sqrt{m_{2\mu}^2 + m_{2\tau}^2 + M^2}}, \\ s_e &\equiv \frac{m_{2e}}{M_D} && \text{and} \quad c_e \equiv \frac{\sqrt{m_{2\mu}^2 + m_{2\tau}^2 + M^2}}{M_D}\,. \end{aligned} \tag{2.67}$$

Then, to separate the N_2' contribution from the active masses three rotations can be made, one for each flavor. Explicitly:

$$\begin{aligned} U_{active} &= R_{34}(\theta_\tau)R_{24}(\theta_\mu)R_{14}(\theta_e) \\ &= \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & c_\tau & s_\tau & 0 \\ 0 & 0 & -s_\tau & c_\tau & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & c_\mu & 0 & s_\mu & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & -s_\mu & 0 & c_\mu & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_e & 0 & 0 & s_e & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ -s_e & 0 & 0 & c_e & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}. \end{aligned} \tag{2.68}$$

After these rotations, the active sector is perfectly diagonalized, leaving only the sterile masses off-diagonal. To diagonalize the 2×2 block, a unitary transformation is needed, such as

$$U_{sterile} = \frac{1}{\sqrt{2}} \begin{pmatrix} -i & 1 \\ i & 1 \end{pmatrix}. \tag{2.69}$$

By embedding $U_{sterile}$ into a 5 dimensional matrix and and taking its product with the active rotation matrix, the final unitary matrix is completely determined within the $U(1)_N$ conserving limit

$$U = \underbrace{\begin{pmatrix} c_e & 0 & 0 & s_e & 0 \\ -s_e s_\mu & c_\mu & 0 & c_e s_\mu & 0 \\ -s_e c_\mu s_\tau & -s_\mu s_\tau & c_\tau & c_e c_\mu s_\tau & 0 \\ -s_e c_\mu c_\tau & -s_\mu c_\tau & -s_\tau & c_e c_\mu c_\tau & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}}_{U_{active}} \underbrace{\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -i/\sqrt{2} & 1/\sqrt{2} \\ 0 & 0 & 0 & i/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}}_{\mathrm{diag}(\mathbb{I}_{3\times 3}, U_{sterile})}, \tag{2.70}$$

and with it, the mixing angles $\boldsymbol{\theta}$ and the sub-matrix $\mathfrak{U}$ can be extracted:

$$\boldsymbol{\theta} = \frac{1}{\sqrt{2}} \begin{pmatrix} i s_e & i c_e s_\mu & i c_e c_\mu s_\tau \\ s_e & c_e s_\mu & c_e c_\mu s_\tau \end{pmatrix} \quad \text{and} \quad \mathfrak{U} = \frac{1}{\sqrt{2}} \begin{pmatrix} -i c_e c_\mu c_\tau & c_e c_\mu c_\tau \\ i & 1 \end{pmatrix}. \tag{2.71}$$

Finally, the primed fields are related to the physical ones as:

$$\begin{aligned} \nu_e' &= \frac{1}{\sqrt{2}} s_e (N_2 - i N_1) + c_e\, \nu_e, \\ \nu_\mu' &= \frac{1}{\sqrt{2}} c_e s_\mu (N_2 - i N_1) - s_e s_\mu \nu_e + c_\mu\, \nu_\mu, \\ \nu_\tau' &= \frac{1}{\sqrt{2}} c_e c_\mu s_\tau (N_2 - i N_1) - s_\tau (s_e c_\mu \nu_e + s_\mu \nu_\mu) + c_\tau\, \nu_\tau, \\ N_1' &= \frac{1}{\sqrt{2}} c_e c_\mu c_\tau (N_2 - i N_1) - c_\tau (s_e c_\mu \nu_e + s_\mu \nu_\mu) + s_\tau\, \nu_\tau, \\ N_2' &= \frac{1}{\sqrt{2}} (N_2 + i N_1). \end{aligned} \tag{2.72}$$

Chapter 3

Introduction to Effective Field Theory and Sterile Dipole Physics

3.1 The Origin of Effective Field Theory

Quantum Field Theory originated in the late 1920s as a solution for the electromagnetic field quantization, and it has become increasingly successful, leading to the formulation of the Standard Model [22]. The theory comes out naturally when imposing the invariance under the Lorentz group and the G_{SM} gauge group, and once a specific particle content and renormalizability are imposed. In particular, this last requirement was considered a fundamental criterion for any viable physical model, since the perturbative expansion of couplings with negative mass dimensions gave infinite and divergent contributions. This was considered a nonsensical result that would have required an infinite number of counterterms to absorb the ultraviolet (UV) divergencies, and therefore such couplings were discarded. Such formalism, however, led to important discrepancies with the observational data and the models that would describe them, as seen in 2. A different solution to those problems was suggested by Kenneth Wilson, changing radically the community approach to renormalizability with respect to its original formulation. In fact, he advanced the idea of considering every physical theory, including the Standard Model, as an Effective Field Theory (EFT) [8]. In this way, any model is expected to accurately describe a phenomenon only within a certain energy range and well below a physical threshold Λ. This was already implicit in the structure of nature as, for example, the computation of a macroscopic object trajectory does not require detailed knowledge of its internal subatomic structure. Similarly, studying the low-energy dynamics of elementary particles does not require the exact knowledge of the physics at the Planck scale. This intuition is then encapsulated in the principle of scale separation. It tells us, indeed, that physical phenomena occurring at vastly different length or energy scales effectively decouple from one another. It becomes necessary, then, to embed a cutoff scale in the model, representing the point at which the effective theory breaks down due to the

introduction of new and unknown degrees of freedom.

In Quantum Field Theory, this concept is formally expressed as the Appelquist-Carazzone decoupling theorem [23].It states that, in a gauge theory involving particles much heavier than the energy scale, they can manifest in only two ways: (i) they can either appear as a shift of coupling and masses of light fields, or (ii) they can appear as local interactions between light fields (effective operators). In the first case, the shift in the parameter space gets absorbed in the measured values for the light particles, causing the presence of the heavy particle to be inherently unobservable. The second manifestation of those heavy degrees of freedom, instead, is strictly suppressed by a factor proportional to $(E/M)^n$, where E is the system energy scale, M the mass of the heavy particles and $n \in \mathbb{N}$ depends on the effective operator dimension. Consequently, the heavy particles decouple from the low-energy theory and can be "integrated-out" from the action. The remaining effective Lagrangian is then composed solely by the light fields and is known as the Infrared (IR) theory.

This decoupling principle leads to two complementary methods in theoretical model building:

- The top-down approach starts from the high-energy Lagrangian completely defined and the low-energy effective theory is recovered by the explicit evaluation of the path integral over the heavy fields in the $E \ll M$ limit. The low-energy couplings are then derived analytically and exactly as functions of the high-energy parameters. A practical example of this procedure is the derivation of the active neutrino effective Majorana mass matrix in the Type-I Seesaw mechanism, obtained by integrating out the heavy sterile singlets N' at tree-level [24, 25];

- The bottom-up approach finds its application in the case of an unknown UV completion, such as the current situation for New Physics beyond the SM. Here, the starting point becomes the low-energy, and fully known, particle content and gauge symmetries to which all possible higher-dimensional operators are added, compatible with the assumed symmetries, encoding the high-energy effects. The new terms in the Lagrangian are multiplied by the Wilson coefficients, whose values are to be determined, and suppressed explicitly by the Λ cutoff scale. Following this procedure a model-independent theory can be built, allowing to test experimental deviations without losing generality [9].

Dimensional analysis is essential to a correct interpretation of an Effective Field Theory and understanding the presence of higher-dimensional operators. Since the action of the physical system,

$$S = \int d^4x \, \mathcal{L}, \tag{3.1}$$

enters in the phase of the path integral, it must have vanishing energy dimensions. So, working in natural units, this constraint forces the Lagrangian to assume $[\mathcal{L}] = 4$ in order to balance the negative dimensions of the integration measure. The dimensions of the fields can now be determined by imposing that their kinetic terms have the same dimension as the Lagrangian. For example, the kinetic term of a scalar field has the form $\partial_\mu \phi \partial^\mu \phi$, implying the field to have $[\phi] = 1$. On the other hand, a fermion field kinetic term involves only one

space time derivative, forcing the spinor field to have $[\psi] = 3/2$. Once defined these canonical dimensions, the most general Lagrangian of an Effective Theory is nothing other than an infinite series of local operators compatible with the system symmetries:

$$\mathcal{L}_{\text{eff}} = \sum_i c_i \mathcal{O}_i \,, \tag{3.2}$$

where the coupling c_i must have the right dimensions in order to absorb the operator ones so that the overall term is compatible with the Lagrangian dimensions. To highlight the energy scale suppression of the operator $\mathcal{O}_i$, its coefficient can be rewritten as

$$c_i = \frac{C_i}{\Lambda^{d_i-4}}, \tag{3.3}$$

where $d_i \equiv [\mathcal{O}_i]$ and C_i is the dimensionless Wilson coefficient.

Moreover, operators are divided into three distinct categories, depending on their dimensions d_i. Each class of operators plays a different role, depending on the energy scale E considered [26]:

$d_i < 4$: Relevant Operators are multiplied by a positive dimension coupling, making their effects scale as $(\Lambda/E)^{4-d_i}$. Therefore, those operators are dominant in the infrared regime, while they become negligible at high-energies. For example, the mass operator $\frac{1}{2}m^2\phi^2$ lives inside this class;

$d_i = 4$: Marginal Operators, such as kinetic terms and Yukawa interactions, have dimensionless couplings, giving them an energy independent contribution;

$d_i > 4$: Irrelevant Operators are matched by negative dimensions couplings, which suppress their contribution as $(E/\Lambda)^{d_i-4}$. A complete and general effective theory should accommodate an infinite number of them, but since their effects are strongly hierarchically ordered, only few of them can enter the Lagrangian, given the required precision $(E/\Lambda)^n$. In fact, once chosen at which order the operators become negligible, the number of the irrelevant operators included in the theory is finite and well-defined.

Historically, theories were built out of only relevant and marginal operators, since irrelevant ones were considered problematic. Models of this kind are called renormalizable theories. Including a higher-dimensional operator, instead, generates ultraviolet divergences, leading to an infinite series of counterterms in order to reabsorb them. Those so called non-renormalizable theories were then characterized by a very low predicting power. For this reason, modern EFT are not required to be valid at all energies, solving the predictivity problem.

3.2 Integrating Out Heavy Fields

The top-down procedure mentioned before consists of extracting a low-energy effective Lagrangian $\mathcal{L}_{eff}$ from a known high-energy theory and in the literature it is called *integrating out* or *matching*. This framework can be demonstrated through an explicit and systematic derivation, starting from the path integral formulation of the theory. This operation consists of performing the functional integration perturbatively over the heavy degrees of freedom, since an exact

analytical integration is generally unfeasible for interacting theories. In the following subsections, we will illustrate this expansion with a detailed computation at the classical tree-level, where the heavy fields are replaced by their classical equations of motion, and at the one-loop level, which introduces quantum fluctuations, divergences, and renormalization.

3.2.1 Tree-Level Matching

The leading-order contribution to the effective action is obtained by evaluating the path integral in the classical limit, commonly referred to as the saddle-point approximation. In this regime, the quantum fluctuations of the heavy fields are strictly neglected, and their dynamics are entirely governed by their classical Euler-Lagrange equations of motion. Integrating out a heavy particle at tree-level therefore translates into solving its equation of motion, expanding the solution in the low-energy limit, and substituting it back into the fundamental Lagrangian.

Consider, for example, a system involving a light fermion field ψ and a heavy scalar field Φ, with masses m and M respectively. The generating functional, defined as the integral over all their dynamical configurations, then reads:

$$\mathcal{Z} = \int \mathcal{D}\psi\, \mathcal{D}\Phi\, \exp\left(i \int d^4x\, \mathcal{L}_{\mathrm{UV}}(\psi, \Phi) \right), \tag{3.4}$$

where the Lagrangian has the form of, for example,

$$\mathcal{L}_{\mathrm{UV}} = i\psi^\dagger \bar{\sigma}^\mu \partial_\mu \psi - \frac{1}{2} m(\psi\psi + \psi^\dagger\psi^\dagger) + \frac{1}{2}\partial_\mu\Phi\partial^\mu\Phi - \frac{1}{2}M^2\Phi^2 - y\Phi(\psi\psi + \psi^\dagger\psi^\dagger), \tag{3.5}$$

where the spinor contraction is defined as $\psi\psi \equiv \varepsilon^{\alpha\beta}\psi_\beta\psi_\alpha$ and y is the dimensionless Yukawa coupling, which can be checked through a dimensional analysis.

Let us suppose, now, to be interested only in the fermion processes, happening at energy scales $E \sim m \ll M$. It is convenient, then, to separate the integrals in the two fields and perform the one over the scalar degrees of freedom. Hence, define the effective theory through the condition:

$$\exp\left(i \int d^4x\, \mathcal{L}_{\mathrm{eff}}(\psi) \right) = \int \mathcal{D}\Phi\, \exp\left(i \int d^4x\, \mathcal{L}_{\mathrm{UV}}(\psi, \Phi) \right). \tag{3.6}$$

Solving this functional integral is generally complex and introduces loop-level quantum corrections. Its tree-level solution, however, can be obtained straightforwardly making use of the saddle-point approximation, which is equivalent to solving the classical Euler-Lagrange Equation of Motion for the heavy field and substituting the solution back into the original action S [8]. Indeed, this approximation claims that

$$e^{iS_{EFT}[\psi]} = e^{iS[\psi, \hat{\Phi}]} \quad \text{such that} \quad \left.\frac{\delta S}{\delta \Phi}\right|_{\hat{\Phi}} = 0. \tag{3.7}$$

The first step is then the derivation of the Equation of Motion

$$\frac{\partial \mathcal{L}_{\mathrm{UV}}}{\partial \Phi} - \partial_\mu \left(\frac{\partial \mathcal{L}_{\mathrm{UV}}}{\partial(\partial_\mu \Phi)} \right) = 0, \tag{3.8}$$

which can be written as:

$$(\Box + M^2)\Phi = -y(\psi\psi + \psi^\dagger\psi^\dagger), \tag{3.9}$$

where $\Box \equiv \partial_\mu\partial^\mu$ is the d'Alembertian operator. Formally, the classical configuration for the scalar field can be achieved by inverting the differential operator,

$$\Phi = -\frac{1}{\Box + M^2} y(\psi\psi + \psi^\dagger\psi^\dagger), \tag{3.10}$$

describing how it responds to the source current $(\psi\psi + \psi^\dagger\psi^\dagger)$ generated by the fermion fields. Inserting now the low-energy limit in the equation, the light particles momenta are required to be small compared to the scalar mass: $p^2 \ll M^2$, causing the d'Alembertian operator to obey the same condition, since $p^2 = (i\partial^\mu)(i\partial_\mu) = -\Box$. This assumption allows us to perform a Taylor expansion of the inverse operator, since the fraction of Eqn. (3.10) is considered small:

$$\frac{1}{\Box + M^2} = \frac{1}{M^2}\frac{1}{1 + \frac{\Box}{M^2}} = \frac{1}{M^2}\left(1 - \frac{\Box}{M^2} + \frac{\Box^2}{M^4} - \dots\right). \tag{3.11}$$

The solution of the Equation of Motion for the scalar field then reads:

$$\Phi \approx -\frac{y}{M^2}(\psi\psi + \psi^\dagger\psi^\dagger) + \frac{y}{M^4}\Box(\psi\psi + \psi^\dagger\psi^\dagger) + \mathcal{O}\left(\frac{1}{M^6}\right). \tag{3.12}$$

Before inserting the solution into the Lagrangian $\mathcal{L}_{UV}$, the kinetic and mass terms of the scalar field can be conveniently combined as

$$\frac{1}{2}\partial_\mu\Phi\partial^\mu\Phi - \frac{1}{2}M^2\Phi^2 = -\frac{1}{2}\Phi(\Box + M^2)\Phi, \tag{3.13}$$

where additional total derivatives have been neglected due to their vanishing contribution once inside the action S. Using Eqn. (3.9), the mass and kinetic term can then be rewritten as:

$$\mathcal{L}_{UV} \supset \frac{1}{2}\Phi\big[y(\psi\psi + \psi^\dagger\psi^\dagger)\big], \tag{3.14}$$

which is now proportional to the Yukawa interaction term. The effective Lagrangian for the light fermions becomes:

$$\begin{aligned}\mathcal{L}_{\text{eff}} &= i\psi^\dagger\bar{\sigma}^\mu\partial_\mu\psi - \frac{1}{2}m(\psi\psi + \psi^\dagger\psi^\dagger) - \frac{1}{2}y\,\Phi(\psi\psi + \psi^\dagger\psi^\dagger) \\ &= i\psi^\dagger\bar{\sigma}^\mu\partial_\mu\psi - \frac{1}{2}m(\psi\psi + \psi^\dagger\psi^\dagger) + \frac{y^2}{2M^2}(\psi\psi + \psi^\dagger\psi^\dagger)^2,\end{aligned} \tag{3.15}$$

where the second step is obtained by inserting the scalar field solution (3.12) to its lowest order.

The new Lagrangian, therefore, replaces the exchange of the scalar particle between light fermions, where Φ had the role of the interaction mediator, with a local four-fermion interaction. Moreover, a new, dimension-six operator $\mathcal{O}_6 \equiv (\psi\psi + \psi^\dagger\psi^\dagger)^2$ has been generated, composed only by the light degrees of freedom. The multiplicative factor $\frac{y^2}{2M^2}$, corresponding to the Wilson coefficient, has mass dimensions equal to -2, absorbing in this way the two extra dimensions $\mathcal{O}_6$.

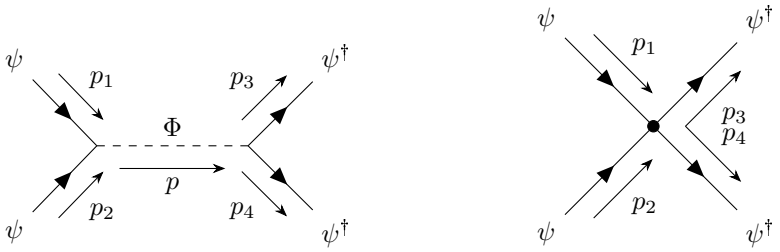


Figure 3.1: Tree-level matching between the high-energy UV theory and the low-energy EFT. To the left: scattering process in the UV theory mediated by a heavy scalar Φ of mass M. To the right: the equivalent process in the EFT ($E \ll M$). The heavy mediator is integrated-out, yielding a local four-fermion contact interaction described by a dimension-six operator.

Here, the mass scale M plays the role of the cutoff Λ described in the previous section.

The list of higher-dimensions operator could be extended indefinitely. Indeed, inserting into the effective Lagrangian also the second order term of the Φ Taylor expansion, proportional to $\Box/M^4$, a dimension-eight operator would appear, heavily suppressed by the power of M. This explicitly demonstrates the Wilsonian power counting where infinite irrelevant operators are generated, but with effects hierarchically ordered by inverse powers of the heavy cutoff scale.

The top-down approach can be used to describe the type I Seesaw mechanism discussed in Section 2.2 by integrating out the heavy right-handed neutrino N' [25]. This generates a new operator suppressed by M_N. Such leading-order operator appears to be, therefore, the dimension-5 Weinberg operator $(l\tilde{\varphi})(l\tilde{\varphi})$ [27], due to the presence of two light lepton doublets and two Higgs doublets in the interaction term. This mechanism represents one portal through which active neutrino masses are generated.

3.2.2 One-Loop Matching and the Hierarchy Problem

The above procedure demonstrates how higher-dimensional operators can be generated directly from the classical equations of motion, when only the tree-level is considered. However, quantum fluctuations force the theory to introduce smaller, loop-level corrections increasing significantly the complexity. This is true also when it is required to integrate out heavy fields propagating in loops. To manage such a problem, one must evaluate Feynman diagrams and make use of renormalization techniques to deal with UV divergences. As an example, here the one-loop correction is presented to the bare mass of a light scalar field, induced by a heavy fermion Ψ, such that $M_\Psi \gg m_\phi$. The Yukawa coupling is, as before,

$$\mathcal{L}_{\text{int}} = -\frac{y}{2}\phi(\Psi\Psi + \Psi^\dagger\Psi^\dagger), \tag{3.16}$$

and the goal is to compute the effective mass correction at energies lower than the fermion mass, by integrating it out. Starting from the scalar self-energy diagram, representing a virtual emission and absorption of the fermion during the scalar propagation in vacuum, it is possible to exploit the low-energy limit

considering a vanishing momentum $p_\phi = 0$. Then, the loop integral over the

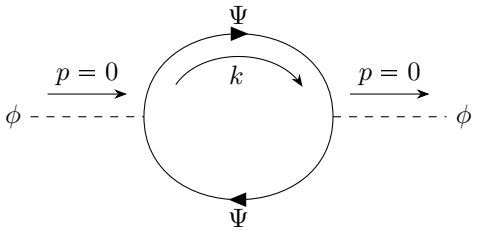


Figure 3.2: One-loop scalar self-energy diagram. The heavy sterile neutrino Ψ runs in the loop, generating a quantum correction to the light scalar mass.

internal fermion momentum k assumes the form:

$$I \propto -y^2 \int \frac{d^4k}{(2\pi)^4} \frac{k^2}{(k^2 - M_\Psi^2)^2}, \tag{3.17}$$

which is quadratically divergent in four spacetime dimensions. In order to avoid divergences, dimensional regularization is necessary. This method continues analytically the spacetime dimensions to $d = 4-2\epsilon$, but to maintain the Yukawa coupling dimensionless an arbitrary mass scale μ must be introduced [28]. Using the master equation for dimensional regularization, the loop integral becomes:

$$I \propto -\frac{y^2 \mu^{2\epsilon}}{16\pi^2} \frac{\Gamma(-1+\epsilon)}{\Gamma(1)} (M_\Psi^2)^{1-\epsilon}, \tag{3.18}$$

where μ is defined as the 't Hooft scale and Γ is the Gamma function. Written in this way, the UV divergences can be isolated through a Taylor expansion of Γ around $\epsilon \to 0$, resulting in

$$I \propto \frac{y^2 M_\Psi^2}{16\pi^2} \left[\frac{1}{\epsilon} + 1 + \ln\left(\frac{4\pi\mu^2}{M_\Psi^2}\right) - \gamma_E \right], \tag{3.19}$$

where γ_E is the Euler-Mascheroni constant, defined as

$$\gamma_E = \lim_{n\to\infty} \left(\sum_{k=1}^{n} \frac{1}{k} - \ln n \right). \tag{3.20}$$

The expression (3.19) presents a simple pole and a finite logarithm, which can both be absorbed along with γ_E through renormalization. In particular the Modified Minimal Subtraction ($\overline{MS}$) scheme is often used, introducing a counterterm explicitly for this purpose [29] and leaving the physical correction to the scalar mass as

$$\Delta m_\phi^2 \simeq \frac{y^2}{16\pi^2} M_\Psi^2 \ln\left(\frac{\mu^2}{M_\Psi^2}\right). \tag{3.21}$$

Concluding, in the low-energy limit the effective physical mass of the scalar field reads:

$$m_{\text{phys}}^2 = m_\phi^2 + \Delta m_\phi^2 \simeq m_\phi^2 + \mathcal{O}\left(\frac{y^2}{16\pi^2} M_\Psi^2\right). \tag{3.22}$$

This result, in the context of the Standard Model, brings to an important theoretical puzzle known as the Hierarchy Problem. In fact, the correction to the scalar mass pushes m^2_{phys} to the highest mass scale available in the theory. To keep the scalar mass as light as initially assumed, however, it would be required an unnaturally precise cancellation between the bare mass m^2_ϕ and the correction itself, since it is not protected by any internal symmetry, as fermion masses are [21]. This becomes particularly important when looking at the Higgs boson, which has $m_h \approx 125$ GeV. Observation of such a mass would require either New Physics to be close to the TeV scale, or an incredible fine-tuning of the Standard Model [8].

3.3 The Standard Model as an Effective Field Theory

The EFT philosophy, thus, considers every theory fully predictive only below a certain energy threshold. It is therefore possible, and in fact provides a robust theoretical framework, to extend this effective approach to the Standard Model itself, treating it as a low-energy approximation of an unknown UV complete theory. This method goes under the name of Standard Model Effective Field Theory (SMEFT) and considers the renormalizable SM Lagrangian $\mathcal{L}^{(4)}_{\text{SM}}$ as only the leading-order term of the theory, containing all possible relevant and marginal operators. The unknown heavy physics is then introduced through a tower of higher dimensional operators, carefully built respecting the SM gauge symmetry group G_{SM} and particle content. Expanding in powers of $1/\Lambda$, the complete Lagrangian takes the form

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}^{(4)}_{\text{SM}} + \frac{1}{\Lambda}\sum_i C^{(5)}_i \mathcal{O}^{(5)}_i + \frac{1}{\Lambda^2}\sum_j C^{(6)}_j \mathcal{O}^{(6)}_j + \mathcal{O}\left(\frac{1}{\Lambda^3}\right), \tag{3.23}$$

where the higher-dimensional operators $\mathcal{O}^{(d)}_i$ are gauge invariant under the group G_{SM} [9]. The New Physics energy scale Λ is still debated, but from the SM phenomenological successes at the electroweak scale ($\sim 10^2$ GeV), it is expected to be significantly higher. However, from experimental evidence such as the existence of active neutrino masses, it is clear that, up to now, the Standard Model is not complete at the renormalizable level. In fact, as discussed in Sec. 2.2, the absence of right-handed singlets for neutrinos in the SM particle content forces them to be strictly massless, requiring thus a correction. With the introduction of these right-handed sterile neutrinos, the necessity of including their contribution in the EFT framework emerges, and it is met by the νSMEFT model [30, 31].

Considering once again the Minimal Seesaw Model presented in 2.3, thus including only two sterile states N'_i, the leading-order Lagrangian for the extended sector reads:

$$\mathcal{L}^{(4)}_N = iN'^{\dagger}_i \bar{\sigma}^\mu \partial_\mu N'_i - \left(y^{\alpha j}_\nu (l^a_\alpha \varepsilon_{ab} \varphi^b) N'_j + \frac{1}{2}(M_N)_{ij} N'_i N'_j + \text{h.c.}\right), \tag{3.24}$$

where the kinetic term and the Yukawa interaction for the HNLs have been included; ε is the antisymmetric tensor acting on the $SU(2)_L$ isospin indices.

After the electroweak symmetry breaking, the Higgs boson φ takes a VEV transforming the Yukawa term into the Dirac mass term for the active neutrinos $m_D \propto y_\nu v$. Through the Seesaw mechanism, this generates an effective Majorana mass for the active neutrinos, which is naturally suppressed by the heavy sterile neutrino mass scale [30, 25]:

$$m_\nu^{seesaw} \sim \frac{y_\nu^2 v^2}{M_N}, \tag{3.25}$$

where $v \simeq 246$ GeV is the Higgs VEV.

The extension $\mathcal{L}_N^{(4)}$, although is capable of providing the generation of active neutrino masses, represents still the leading-order of a complete UV theory taking into account heavy (yet unknown) dynamics at higher energies. Therefore, including the HNLs into the EFT paradigm, the effective Lagrangian must contain all possible gauge invariant irrelevant operators built out from the extended SM particle content. The result consists of the following expansion, defining the νSMEFT model:

$$\mathcal{L}_{\nu SMEFT} = \mathcal{L}_{SM}^{(4)} + \mathcal{L}_N^{(4)} + \frac{1}{\Lambda}\mathcal{L}^{(5)} + \frac{1}{\Lambda^2}\mathcal{L}^{(6)} + \mathcal{O}\left(\frac{1}{\Lambda^3}\right), \tag{3.26}$$

where each $\mathcal{L}^{(d)} = \sum_k C_k^{(d)} \mathcal{O}_k^{(d)}$ encodes the new interactions and $C_k^{(d)}$ represent the dimensionless Wilson coefficients. The νSMEFT approach turns out to be independent of the specific UV model, integrating out the effects of the heavy and unknown degrees of freedom. Higher dimensional operators encapsulate the new physics, radically modifying sterile neutrino phenomenology and opening new production and decay mechanisms [32]. In this way, they are able to probe further phenomenological features on the electromagnetic properties of neutral fermions. Indeed, the vanishing neutrino electric charge does not prevent them from exhibiting effective electromagnetic multipole moments induced by quantum loops mediated by charged particles [33].

3.4 The Leading-Order Operators: Dimension Five

Because operators of dimension d are suppressed by a factor of $1/\Lambda^{d-4}$, the relative importance of higher-dimensional operators is strictly governed by power counting. The leading-order corrections to the theory appear naturally at lowest achievable dimension, after requiring their gauge invariance under the symmetry group. Filtering out irrelevant operators in this way, the class of compatible ones at dimension five is reduced to only three possible cases. These operators are the ones that parametrize the dominant new physics effects at low-energy scales $E \ll \Lambda$ and they are the Weinberg Operator $\mathcal{O}_W^{(5)}$, the Higgs-Sterile Operator $\mathcal{O}_{NH}^{(5)}$ and the Sterile Dipole Operator $\mathcal{O}_{NB}^{(5)}$ [30]. In the following, every operator will be discussed in detail.

3.4.1 The Weinberg Operator ($\mathcal{O}_W^{(5)}$)

The Weinberg operator is a specific combination of only the lepton doublet l and the Higgs doublet φ, initially formulated by Weinberg in 1979 to explain

the origin of neutrino masses [27]:

$$\mathcal{O}_W^{(5)} = C_W^{\alpha\beta}(l_\alpha\tilde{\varphi})(l_\beta\tilde{\varphi}) + \text{h.c.}, \tag{3.27}$$

where α, β are the flavor indices, $\tilde{\varphi}_b = \varepsilon_{bc}\varphi^c$ is the conjugate doublet of the scalar Higgs field and C_W is the symmetric matrix of Wilson coefficients. This operator obtains a relevant role after the electroweak symmetry breaking, when $\langle\varphi\rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}$ and $\mathcal{O}_W^{(5)}$ generates directly a Majorana mass term for the active neutrinos:

$$m_\nu^{\alpha\beta} \simeq C_W^{\alpha\beta}\frac{v^2}{\Lambda}. \tag{3.28}$$

While violating explicitly the lepton number symmetry by two units $\Delta L = 2$, this effective interaction contributes with the Seesaw produced active mass expressed in Eqn (3.25) to the total left-handed neutrino mass. Depending on the sterile mass splitting $\Delta M = M_1 - M_2$, the additive term coming from the Weinberg Operator turns out to be suppressed but proportional to the tree-level mass matrix (in case of degenerate HNLs) or, when $\Delta M \neq 0$, even greater than m_ν^{seesaw} [30]. Moreover, considering the sterile neutrinos at the electroweak scale, the Wilson coefficient C_W could be generated by integrating out heavy fields different from N'. This would allow to decouple the generation of active neutrino mass from the HNL phenomenology, effectively relaxing the constraints on the sterile neutrino interactions.

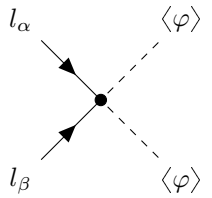


Figure 3.3: Feynman diagram representing the effect of the dimension-five Weinberg operator $\mathcal{O}_W^{(5)}$ subsequent to Electroweak Symmetry Breaking. The clashing fermion arrows explicitly denote the violation of total lepton number by two units ($\Delta L = 2$). The two insertions of the Higgs vacuum expectation value (represented by the dashed lines terminating in $\langle\varphi\rangle$) effectively collapse the original four-point interaction, yielding a local Majorana mass term for the active left-handed lepton doublets l_α and l_β.

3.4.2 The Higgs-Sterile Operator ($\mathcal{O}_{NH}^{(5)}$)

A second dimension five operator is the Higgs-Sterile operator, which couples the Higgs scalar φ to the sterile neutrinos N' [34]:

$$\mathcal{O}_{NH}^{(5)} = C_{NH}^{ij}(N_i'N_j')(\varphi^\dagger\varphi) + \text{h.c.}\,, \tag{3.29}$$

where C_{NH} is a symmetric matrix in flavor space. From dimensional analysis, $[N'] = 3/2$, $[\varphi] = 1$, so the described interaction is at $d = 5$ and also gauge invariant due to the singlet nature of the HNLs, as expected. However, as with

for the Weinberg operator, also $\mathcal{O}_{NH}^{(5)}$ violates the lepton number by two units due to the bilinear $N_i' N_j'$ [30].

Two relevant effects arise from $\mathcal{O}_{NH}^{(5)}$ when expanding the Higgs field around its vacuum expectation value:

$$\varphi \to \langle \varphi \rangle + \begin{pmatrix} 0 \\ \frac{h}{\sqrt{2}} \end{pmatrix}. \tag{3.30}$$

Firstly, a direct, dimension five, contribution to the sterile Majorana mass matrix originates from the vacuum insertion, inducing a constant shift:

$$\tilde{M}_N \simeq M_N - C_{NH} \frac{v^2}{\Lambda}. \tag{3.31}$$

The contribution of the Higgs-sterile interaction to the mass matrix yields a constraint for the New Physics scale. In fact, to ensure the renormalizable contribution as the leading-order, a suppression of the effective term by $\Lambda \gtrsim 60 \text{ TeV} \times (1 \text{ GeV}/M_N)$ is required [31]. A UV completion with smaller cutoff, or under the Minimal Flavor Violation (MFV) hypothesis, the Wilson coefficient C_{NH} scales proportionally to M_N/Λ, meaning that at first order no suppression from the neutrino Yukawa coupling y_ν is present. Without this suppression, $\mathcal{O}_{NH}^{(5)}$ represents the dominant operator for the sterile neutrino production at colliders [30].

A second and important consequence of this dimension five interaction is the introduction of a new vertex that couples the Higgs boson h with a pair of sterile neutrinos:

$$\mathcal{L}_{hNN} = \frac{v}{\Lambda} C_{NH}^{ij} \, h \, (N_i' N_j') + \text{h.c.}\,. \tag{3.32}$$

This vertex would represent an exotic Higgs decay into the pair of heavy neutrinos. Considering the diagonal case with $i = j$, its partial decay width is given by, when the kinematic threshold $m_h > 2m_N$ is met [34]:

$$\Gamma(h \to N_i N_i) = \frac{1}{2\pi} \frac{v^2}{\Lambda^2} m_h \beta_N^3 (C_{NH}^{ii})^2, \tag{3.33}$$

where m_h is the Higgs mass and $\beta_N = \sqrt{1 - \frac{4m_N^2}{m_h^2}}$.

This operator, thus, introduces a new portal for the production for sterile neutrinos through the so-called Higgs-Strahlung. This mechanism would make use of both processes

$$e^+ e^- \to Zh \quad \text{and} \quad h \to NN, \tag{3.34}$$

and its cross section depends exclusively on the exotic branching ratio $\text{BR}(h \to NN)$. Their production, therefore, would not be regulated nor suppressed by the mixing angles $\theta_{\alpha i}$ between active and sterile neutrinos in any way, allowing them to be produced even in the limit of extremely small mixing conditions [34]. Depending on the active-sterile mixing, the HNLs lifetime can be divided in three different regimes for the detectors: (i) prompt decays, where up to six fermions are expected in the final state; (ii) displaced decays, which reduce drastically the SM background; and (iii) decays outside the detectors, if the sterile neutrinos are stable enough. Each one of these regimes sets different constraints to the New Physics scale [34].

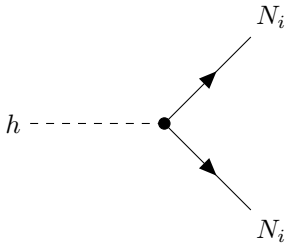


Figure 3.4: Feynman diagram illustrating the exotic decay of the physical Higgs boson h into a pair of identical sterile neutrinos (N_i), mediated by the dimension-five operator $\mathcal{O}_{NH}^{(5)}$. The diverging fermion arrows explicitly reflect the Majorana nature of the vertex and the associated lepton number violation. This effective interaction, whose coupling scales as v/Λ following Electroweak Symmetry Breaking, provides a critical, mixing-independent production portal for Heavy Neutral Leptons at collider facilities.

3.4.3 The Sterile Dipole Operator ($\mathcal{O}_{NB}^{(5)}$)

The last gauge invariant operator at dimension five couples directly the sterile fields with the $U(1)_Y$ gauge boson $B_{\mu\nu}$, the hypercharge field strength tensor. The operator has the form:

$$\mathcal{O}_{NB}^{(5)} = C_{NB}^{ij} N_i' \sigma^{\mu\nu} N_j' B_{\mu\nu} + \text{h.c.}, \tag{3.35}$$

where $B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$ and $\sigma^{\mu\nu} = \frac{i}{4}(\sigma^\mu \bar{\sigma}^\nu - \sigma^\nu \bar{\sigma}^\mu)$ is the antisymmetric tensor and $\sigma^\mu = (\mathbb{1}, \sigma^i)$, $\bar{\sigma}^\mu = (\mathbb{1}, -\sigma^i)$ are the Pauli matrices embedded in the four-dimensional spacetime ($\mu = 0, 1, 2, 3$). Unlike the other dimension five operators presented, the properties of the Wilson coefficient C_{NB} here are constrained by the presence of $\sigma^{\mu\nu}$. In fact, due to its antisymmetry, also the fermion operator is antisymmetric under exchange of indices:

$$N_i' \sigma^{\mu\nu} N_j' = -N_j' \sigma^{\mu\nu} N_i' \,, \tag{3.36}$$

implying the same property on the coefficient, thus $C_{NB}^{ij} = -C_{NB}^{ji}$. Consequently its diagonal elements are identically zero, preventing any sterile self-coupling and requiring therefore at least two HNL fields for the operator to be generated [35].

At energies lower than the electroweak scale, the hypercharge boson B_μ can be decomposed into the photon A_μ and the Z_μ boson, following the definition

$$B_\mu = \cos\theta_W A_\mu - \sin\theta_W Z_\mu, \tag{3.37}$$

where θ_W is the weak mixing angle. The symmetry breaking, hence, causes the operator $\mathcal{O}_{NB}^{(5)}$ to couple the two sterile neutrinos with both a photon and a Z boson, inducing two decay channels:

$$\begin{aligned} N_i &\to N_j \, \gamma, \\ N_i &\to N_j \, Z. \end{aligned} \tag{3.38}$$

Further properties of the Dipole Operator and the important consequences of its presence into the effective Lagrangian will be discussed in depth in Sec. 3.6

3.5 Low-Energy Dominance and Dimension-6 Operators

The leading-order correction to the renormalizable theory, the dimension five Lagrangian, thus contains three operators, but to precisely isolate the electromagnetic properties of the sterile sector, further filtering is required.

In fact, the effects of the Weinberg Operators $\mathcal{O}_W^{(5)}$ are entirely absorbed within the generation of the active neutrino masses shown in Eqn. (3.28), while at the same time it does not couple with the sterile sector. At low-energies, this operator collapses into an active Majorana mass correction to the Type-I seesaw effective mass in the flavor basis. Thus, during diagonalization, the contribution of C_W coefficient gets completely absorbed into the physical observable masses, leaving no signatures to be detected. Furthermore, no other effects are produced by this operator, especially in the sterile sector since they are not directly coupled together. Hence, the Weinberg Operator is not able to interact with the sterile neutrinos, nor to mediate their production or decay portals within the colliders, allowing to neglect it for the following discussion.

The Higgs-Sterile Operator $\mathcal{O}_{NH}^{(5)}$, instead, has a relevant role in the collider observations due to the exotic Higgs decay ($h \to NN$) that provides a direct sterile production mechanism. However, because of its explicit lepton number violation, the Wilson coefficient C_{NH} results heavily suppressed. This allow us to consider the effective contribution of the operator negligible with respect to the lepton number conserving interactions or the ones independent from the mixing. As for the Weinberg Operator, then, also the Higgs-Sterile one will be neglected in the following, leaving only the Dipole Operator $\mathcal{O}_{NB}^{(5)}$ to govern the low-energy phenomenology and to describe the HNLs electromagnetic properties and gauge interactions.

EFT, thus, provides a tower of corrections to the renormalizable theory where each order is suppressed by a factor of $(E/\Lambda)^{d-4}$, depending on its dimension d [8]. Hence, due to the value of the cutoff Λ, as the operator dimensions increase their suppression becomes stronger at energies $E \ll \Lambda$, establishing a clear hierarchy among them. Consequently, lower-dimension irrelevant operators, especially the ones at dimension five when available in the theory, dominate over higher order corrections. Therefore, dimension-six and higher dimensional operator can be neglected, unless they generate new vertices not yet present in the Lagrangian. Considering dimension-six operators in the effective sterile Lagrangian would then cause an unmanageable proliferation of unknown, strongly suppressed, Wilson coefficients whose analysis goes beyond the scope of this discussion. Nevertheless, here some of these negligible operators are presented briefly, showing the new, broader and more complex interaction topologies they are able to introduce and would be forbidden at lower mass dimensions. They can be classified into two different categories based on their field content and gauge structure: (i) operators coupling the sterile sector to the SM bosons and (ii) four-fermion contact interactions.

In the first case, new couplings are introduced between HNLs and the Higgs boson, together with the electroweak gauge bosons. Examples include active-sterile transition dipoles

$$\mathcal{O}_{LNB}^{6} \propto (l\sigma^{\mu\nu}N')\tilde{\varphi}B_{\mu\nu} \quad \text{and} \quad \mathcal{O}_{LNW}^{6} \propto (l_a\sigma^{\mu\nu}N')(\sigma_I)_b^a\tilde{\varphi}^b W_{\mu\nu}^I, \tag{3.39}$$

where $I = (1, 2, 3)$ is the isospin index. Those operators induce the sterile singlet radiative decays: $N \to \nu\gamma$ and $N \to \nu Z$. Also dimension-six corrections to the Yukawa coupling

$$\mathcal{O}^6_{LNH} \propto (l\tilde{\varphi})N(\varphi^\dagger\varphi) \tag{3.40}$$

are included, causing the active-sterile mixing parameters to be altered [36].

The second class of operators includes all possible combinations coupling two sterile fields with a pair of fermions, both quarks or leptons. They typically assume a form such as

$$\mathcal{O}^6_{NNee} \propto (N'^\dagger \bar{\sigma}^\mu N')(e^{c\dagger}\bar{\sigma}_\mu e^c) \quad \text{or} \quad \mathcal{O}^6_{N'N'll} \propto (N'^\dagger \bar{\sigma}^\mu N')(l^\dagger \bar{\sigma}_\mu l). \tag{3.41}$$

These operators hold a particularly interesting role for collider experiments since they connect the neutral sterile current to the standard matter one, providing new production mechanisms not suppressed by mixing parameters, such as $e^+e^- \to NN$. Along with new the production portal, three-body decay channels also open, allowing processes of the type $N \to 3f$, where f is any SM fermion [37].

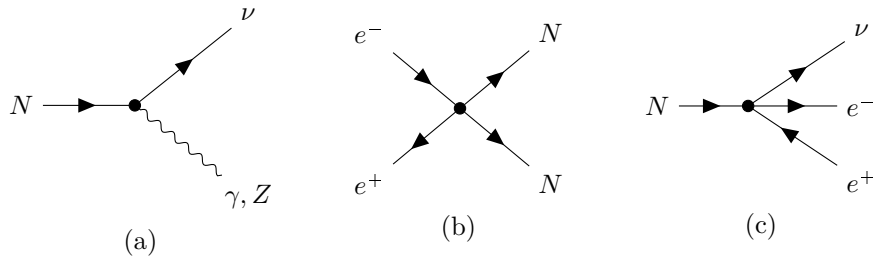


Figure 3.5: Phenomenological implications and novel interaction topologies introduced by dimension-six operators within the νSMEFT framework. **(a)** Radiative decay of the sterile state ($N \to \nu\gamma$ or $N \to \nu Z$) mediated by the active-sterile transition dipole operators $\mathcal{O}^6_{LNB}$ and $\mathcal{O}^6_{LNW}$ following electroweak symmetry breaking. **(b)** Exotic pair-production mechanism at colliders (e.g., $e^+e^- \to NN$) governed by four-fermion neutral current contact interactions ($\mathcal{O}^6_{NNee}$), representing a crucial production portal unsuppressed by the active-sterile mixing angle. **(c)** Three-body decay topology ($N \to 3f$, illustrated here as $N \to \nu e^+e^-$) enabled by four-fermion operators.

3.6 The Sterile Dipole Operator

The Effective Sterile Lagrangian, as previously demonstrated, at dimension five accommodates only the sterile dipole operator, making it particularly interesting when studying HNLs phenomenology. Here a comprehensive theoretical and phenomenological analysis is presented. The gauge invariant operator reads:

$$\mathcal{O}^{(5)}_{NB} = C^{ij}_{NB} N'_i \sigma^{\mu\nu} N'_j B_{\mu\nu} + \text{h.c.}. \tag{3.42}$$

It is interesting to notice how the sterile-sterile coupling is not the only interaction allowed that can be described with the dipole operator. In fact, below the

electroweak scale, also the active-sterile structure respects the symmetry under the SM gauge group and thus should be included in the general effective Lagrangian. This operator is known as the transition dipole moment and it couples an active neutrino ν to the heavy sterile state N' and to the electromagnetic field strength tensor $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. This interaction allows therefore the emission or absorption of a photon between the two fermions. The effective Lagrangian term, at low-energies, reads:

$$\mathcal{L}_{\nu N}^{\text{low-E}} \propto \frac{C_{\nu N}}{\Lambda}\, \nu \sigma^{\mu\nu} N' F_{\mu\nu} + \text{h.c.}\,, \tag{3.43}$$

where Λ is the theory cutoff scale. Through dimensional analysis, then, the operator appears to have a mass dimension of $d = 5$, since $[\nu] = [N'] = 3/2$ and $[F_{\mu\nu}] = 2$. The transition dipole operator written in this way, however, is no longer valid once energies close to or greater than the electroweak symmetry breaking scale are considered. In fact, at those energies, the neutrino field is part of the l doublet, while the $F_{\mu\nu}$ tensor is a linear combination of the gauge tensors $B_{\mu\nu}$ and $W^3_{\mu\nu}$, acting on $U(1)_Y$ and $SU(2)_L$, respectively. At the same time, the more straightforward bilinear, $l\sigma^{\mu\nu}N'B_{\mu\nu}$, breaks explicitly $SU(2)_L$ invariance and must then be excluded from the Lagrangian. A fully gauge-invariant operator must, therefore, contract the lepton doublet indices with a scalar field, more precisely the Higgs doublet φ. Introducing this new field, however, also an extra dimension is added, forcing the leading-order operator to increase to $d = 6$ [38]:

$$\mathcal{L}_{\nu N}^{(6)} = \frac{C^B_{\nu N}}{\Lambda^2}(\epsilon_{ab} l^a \varphi^b)\sigma^{\mu\nu} N' B_{\mu\nu} + \frac{C^W_{\nu N}}{\Lambda^2}(l^a (\sigma_I)_a{}^b \tilde{\varphi}_b)\sigma^{\mu\nu} N' W^I_{\mu\nu} + \text{h.c.}\,, \tag{3.44}$$

where σ_I are the Pauli matrices associated with $SU(2)_L$ and a, b are the isospin indices. Returning now to the low-energy limit, the Higgs field takes a vacuum expectation value $\langle\varphi\rangle = \frac{1}{\sqrt{2}}(0, v)^T$ and the electroweak symmetry gets broken, reducing in this way the operator dimensions back to $d = 5$:

$$\mathcal{L}_{\nu N}^{\text{low-E}} \propto \frac{v}{\Lambda^2} \nu \sigma^{\mu\nu} N' F_{\mu\nu} + \text{h.c.}\,, \tag{3.45}$$

where the electromagnetic field strength comes from the projection of the gauge fields $B_{\mu\nu}$ and $W^3_{\mu\nu}$. This derivation demonstrates that the effective scale of the transition dipole moment is heavily suppressed by the ratio v/Λ^2 and therefore can be neglected when compared to the sterile-sterile dipole operator.

The sterile-sterile dipole moment is expected to become particularly relevant in the chosen mass scale for the sterile neutrinos 0.1 GeV $\lesssim M_N \lesssim$ 10 GeV changing significantly the phenomenology [37, 10]. Furthermore, despite the transition dipole moment being neglected, still an active-sterile dipole coupling must be included in the theory, generated by the mixing.

3.6.1 One-Loop Dipole Generation

A more detailed study of $\mathcal{O}^{(5)}_{NB}$ and of its Wilson coefficient, would reveal the impossibility to generate it at tree-level [34]. Its loop-level-only generation has its roots in two different reasons. In order to produce an effective operator at tree-level, in fact, a Feynman diagram has to describe the operator vertex,

making use at most of one mediator, representing the integrated-out heavy field of the UV completion[1]. However, within the dipole operator, the gauge boson $B_{\mu\nu}$ has to couple to two hypercharge-singlet fields, making any vertex impossible even when a mediator is considered. A second motivation is the dimension of the vertex itself, which is proportional to the boson transferred momentum. Indeed, a renormalizable theory would require vertices with mass dimension $d \leqslant 4$. In the case of a three-particles interaction, the only admitted structure has the form of

$$\mathcal{L}_{UV} \supset g\psi^{\dagger}\bar{\sigma}^{\mu}\psi B_{\mu}, \tag{3.46}$$

which has exactly dimension four. By introducing in the coupling the boson $B_{\mu\nu} = \partial_{\mu}B_{\nu} - \partial_{\nu}B_{\mu}$, instead, also a derivative is introduced, which in momentum space translates as the transferred momentum, adding an extra dimension to the vertex. In conclusion, no vertex for the dipole operator is available at tree-level and a loop-level analysis is required. To avoid the issues described above, the UV theory completion has to accommodate two heavy fields with non-vanishing hypercharge, such as a fermion Ψ and a scalar Φ, to run in the quantum loop as shown in Fig. 3.6. Hence the dipole operator $\mathcal{O}^{(5)}_{NB}$ emerges once the heavy degrees of freedom are integrated-out from the Lagrangian.

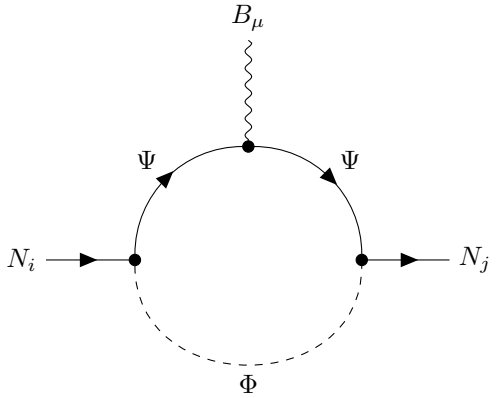


Figure 3.6: Feynman diagram illustrating the one-loop generation of the effective dipole operator $\mathcal{O}^{(5)}_{NB}$. The flavor transition between the sterile neutrino states N_i and N_j is mediated by heavy degrees of freedom residing in the ultraviolet (UV) completion of the theory. Specifically, the loop is formed by a heavy fermion Ψ and a heavy scalar Φ, both possessing non-zero hypercharge. The $U(1)_Y$ gauge boson B_μ couples to the charged internal propagators (shown here interacting with the heavy fermion line, though an analogous diagram exists for the scalar emission). At energy scales well below the heavy masses (Λ), these internal states are integrated-out, yielding the local dimension-five dipole interaction.

From the loop diagram, it is possible to determine the process amplitude $\mathcal{A}_{UV}$ by using the Feynman rules on the three loop propagators. After proper

[1]The Feynman diagram containing the heavy mediator corresponds to the UV completion description. Once the fields are integrated-out at tree-level, the interaction in the diagram becomes a point-like vertex.

computations, the matching condition fixes unambiguously the dipole Wilson coefficient to:

$$C_{NB} = g_Y \frac{g_{UV}^2}{16\pi^2}, \tag{3.47}$$

where g_Y is the hypercharge gauge coupling and g_{UV} corresponds to the unknown coupling describing then the interactions between the heavy fields Ψ, Φ in the loop and the sterile fields N. This result shows that a further suppression is imposed on the dipole operator contribution itself due to the loop prefactor $1/(16\pi^2) \simeq 1/158$ [39]. The dimension-five Lagrangian can then be written as:

$$\mathcal{L}^{(5)}_{dipole} \supset \frac{g_Y}{16\pi^2} \frac{g_{UV}^2}{\Lambda} \left(N_i' \sigma^{\mu\nu} N_j'\right) B_{\mu\nu} + h.c., \tag{3.48}$$

and all the prefactor constants can now be enclosed in a single coupling describing the dipole operator vertex

$$d = \frac{g_Y}{16\pi^2} \frac{g_{UV}^2}{\Lambda}. \tag{3.49}$$

The renormalized, low-energy vertex is then described as in Fig. 3.7. The parametrization of the dipole vertex coupling points out the possibility for two different regimes. Depending on whether the UV coupling remains well below the perturbative limit $g_{UV} \lesssim 1$ or it can reach the non-perturbative limit $g_{UV} \sim 4\pi$, a weakly coupled and suppressed UV completion or a strongly interacting one, where the suppression factor is partially compensated, is expected [40].

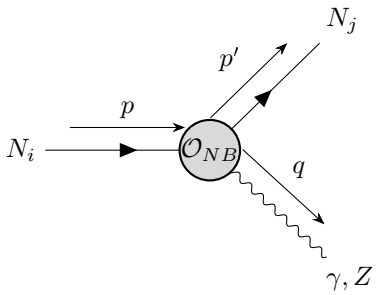


Figure 3.7: Effective Feynman diagram illustrating the dimension-five transition dipole moment between two sterile neutrino mass eigenstates, N_i and N_j. The gray circle denotes the effective local vertex $\mathcal{O}^{(5)}_{NB}$, which encapsulates the loop-induced UV dynamics integrated-out at the scale Λ. Following Electroweak Symmetry Breaking, the operator mediates the transition by transferring momentum $q = p - p'$ to the physical neutral gauge bosons, γ or Z

3.6.2 Parametrization and Symmetries

The parametrization of the Wilson coefficient d of Eqn. (3.49) shows clearly the power counting that must be satisfied and its suppression. However, unlike the sterile mass matrix, the dipole coupling is not constrained by any symmetry of the sterile sector, shown in Sec. 2.3. In fact, introducing the doublet notation

$\mathbf{N'} \equiv \begin{pmatrix} N_1' \\ N_2' \end{pmatrix}$ the dipole operator reads:

$$\mathcal{L}_{dipole} = \frac{d}{2}\mathbf{N}'_a \sigma^{\mu\nu} \varepsilon^{ab} \mathbf{N}'_b B_{\mu\nu}, \tag{3.50}$$

where the Levi-Civita symbol is needed to maintain the antisymmetric structure in the sterile flavor basis. Hence, this term is invariant under $SU(2)_N$ since $(V_N)^c_a (V_N)^d_b \varepsilon^{ab} = \det(V)\varepsilon^{cd} = \varepsilon^{cd}$, but it breaks $U(1)_N$ symmetry, which transformation would give it a phase of $e^{2i\alpha_N}$, where those symmetries act on the sterile doublet as [38]:

$$\begin{aligned} SU(2)_N: &\quad \mathbf{N'}_a \to (V_N)^b_a \mathbf{N'}_b, \\ U(1)_N: &\quad \mathbf{N'} \to e^{i\alpha_N} \mathbf{N'}. \end{aligned} \tag{3.51}$$

Since the dipole term preserves $SU(2)_N$ without any further requirements and does not affect the sterile mass in any way, the coefficient d can thus be as large as the combination of g_{UV} and Λ allows[2]. Note that a bigger dipole coefficient is desirable because stronger couplings give rise to higher probability of radiative decay and with it, also HNLs detection would be made easier [11, 41]. To be more precise, in the most general Lagrangian, d must be considered a complex coefficient, introducing in this way two parameters instead of one. While the power counting reasoning applies to the modulus $|d|$, no further restrictions are imposed, leaving its phase completely free. Nevertheless, in the following the case that maximizes meson and neutrino decays will be considered, fixing thus the dipole as pure imaginary: $d = |d|e^{i\frac{\pi}{2}}$.

3.6.3 The Dipole Operator in the Physical Mass Basis

To determine the sterile neutrinos radiative decays induced by the dipole operator, the Lagrangian written in terms of the physical fields is required. To achieve this, the same procedure as in Sec. 2.3.1 is applied, starting from the easier case of only one lepton flavor coupling with the sterile sector. This assumption enters in the derivation determining the dimension of the unitary matrix U described in Eqn. (2.48). The exact relation between the physical fields and the primed ones is presented in Eqn. (2.53). Thus, applying the rotation on the fields in the dimension-five operator, it reads:

$$\begin{aligned} \mathcal{L}_{\text{dipole}} &= \frac{1}{2} d \left(\mathfrak{U}\,\mathbf{N} - \frac{\mathfrak{U}\boldsymbol{\theta}\nu}{\sqrt{1-\boldsymbol{\theta}^\dagger\boldsymbol{\theta}}} \right)^T \sigma^{\mu\nu}\varepsilon \left(\mathfrak{U}\,\mathbf{N} - \frac{\mathfrak{U}\,\boldsymbol{\theta}\,\nu}{\sqrt{1-\boldsymbol{\theta}^\dagger\boldsymbol{\theta}}} \right) B_{\mu\nu} + h.c. \\ &= \frac{1}{2} d \left(\mathbf{N}^T - \frac{\nu^T \boldsymbol{\theta}^T}{\sqrt{1-\boldsymbol{\theta}^\dagger\boldsymbol{\theta}}} \right) \mathfrak{U}^T \sigma^{\mu\nu}\varepsilon\,\mathfrak{U} \left(\mathbf{N} - \frac{\boldsymbol{\theta}\,\nu}{\sqrt{1-\boldsymbol{\theta}^\dagger\boldsymbol{\theta}}} \right) B_{\mu\nu} + h.c.\,. \end{aligned} \tag{3.52}$$

[2]Remembering that $\Lambda > 1$ TeV is considered and perturbativity limits $g_{UV} \lesssim 4\pi$.

Using again the property $\mathfrak{U}^T\varepsilon\,\mathfrak{U} = \det(\mathfrak{U})\varepsilon$ of the Levi-Civita tensor, it follows that:

$$\begin{aligned}\mathcal{L}_{\text{dipole}} &= \frac{1}{2}de^{i\phi}\left[2\sqrt{1-\boldsymbol{\theta}^\dagger\boldsymbol{\theta}}\,N_1\sigma^{\mu\nu}N_2B_{\mu\nu} - \nu\boldsymbol{\theta}_a\sigma^{\mu\nu}\varepsilon^{ab}\mathbf{N}_bB_{\mu\nu} - \mathbf{N}_a\sigma^{\mu\nu}\varepsilon^{ab}\boldsymbol{\theta}_b\nu B_{\mu\nu}\right] + h.c.\\ &= de^{i\phi}\left[\sqrt{1-\boldsymbol{\theta}^\dagger\boldsymbol{\theta}}\,N_1\sigma^{\mu\nu}N_2B_{\mu\nu} - \nu\theta_1\sigma^{\mu\nu}N_2B_{\mu\nu} + \nu\theta_2\sigma^{\mu\nu}N_1B_{\mu\nu}\right] + h.c.\,,\end{aligned} \tag{3.53}$$

where the $\mathfrak{U}$ matrix determinant has been computed using the solution in Eqn. (2.49) and the identity $\det(\mathfrak{U}^\dagger\mathfrak{U}) = |\det(\mathfrak{U})|^2$. Explicitly,

$$\det\left(\mathbb{1} - \boldsymbol{\theta}\boldsymbol{\theta}^\dagger\right) = 1 - \boldsymbol{\theta}^\dagger\boldsymbol{\theta}, \tag{3.54}$$

from which it follows that

$$\det(\mathfrak{U}) = e^{i\phi}\sqrt{1-\boldsymbol{\theta}^\dagger\boldsymbol{\theta}}. \tag{3.55}$$

From Eqn (3.53) it is still clearly visible the antisymmetric nature of the operator preventing an active-active dipole coupling [33]. At the same time we can notice that the active-sterile mixing enters into the dipole Lagrangian through the fields rotation, opening a second portal between the two sectors [11]. Since the mixing angles are expected to be small, all quadratic terms in $\boldsymbol{\theta}$ will be neglected in the following, therefore the approximation $\sqrt{1-\boldsymbol{\theta}^\dagger\boldsymbol{\theta}} \simeq 1$ is taken.

That said, introducing a more compact, four-component spinor notation is convenient to express the dipole term, in such a way to facilitate decay widths computations. To build a four-component Dirac fermion for both the sterile and the active physical neutrinos and consistent with our current notation, the Ref. [16] conventions have been used and can be found in its Appendix G. The first step to introduce the new Dirac spinor requires the definition of two degenerate Majorana fermions, Ψ_1, Ψ_2, defined as:

$$\begin{aligned}\Psi_1 &= N_1 + iN_2\\ \Psi_2 &= N_1 - iN_2\end{aligned} \quad\longleftrightarrow\quad \begin{aligned}N_1 &= \frac{\Psi_1+\Psi_2}{2},\\ N_2 &= \frac{\Psi_1-\Psi_2}{2i}\end{aligned} \tag{3.56}$$

such that they satisfy the relation $\Psi_1\Psi_2 = N_1N_1 + N_2N_2$. This is a fundamental and necessary step to ensure consistency with the whole Lagrangian at any time and, in particular, with the mass term to guarantee that the fields still describe the same particles. Then, the sterile mass term, in the physical basis reads:

$$\mathcal{L}_m = M_D(N_1N_1 + N_2N_2) + h.c. = M_D\Psi_1\Psi_2 + h.c.\,, \tag{3.57}$$

where $M_D = \sqrt{M^2 + m_{D2}^2}$ is the mass eigenvalue obtained after the mass matrix $\mathcal{M}$ diagonalization. This result proves that the two sterile combinations have the same mass, allowing the construction of a single four-component Dirac spinor[3]

$$\boldsymbol{\Psi} = \begin{pmatrix}\Psi_1\\ \Psi_2^\dagger\end{pmatrix} = \begin{pmatrix}cN_1' + s\nu'\\ N_2'^\dagger\end{pmatrix}, \tag{3.58}$$

with which the mass term can now be written as $\mathcal{L}_m = M_D\overline{\boldsymbol{\Psi}}\boldsymbol{\Psi}$, where $\overline{\boldsymbol{\Psi}} = \boldsymbol{\Psi}^\dagger\gamma^0 = (\Psi_2, \Psi_1^\dagger)$[4]. Obviously, the same must be true for the active neutrino

[3]The second step of the equivalence is obtained by substituting the result of Eqn. (2.59) into the definition of $\Psi_{1,2}$.

[4]Here, the Gamma Matrices γ^μ in the chiral basis will be always considered

mass term, which is expected to read, in the four-component mass basis, as $\mathcal{L}_m = m_\nu \bar{\nu}_L \nu_L$, where $m_\nu = 0$ from the mass matrix diagonalization. Following the same procedure, the four-component Dirac fermion representing the active neutrino results:

$$\nu_L = \begin{pmatrix} c\,\nu' - s N_1' \\ 0 \end{pmatrix}, \tag{3.59}$$

where the vanishing second component is necessary to preserve the left-handed nature of the field. It is now possible to translate also the dipole term from the two-component to the four-component spinor notation. Starting again from the primed fields and using the Eqn. (2.59) solutions, the bilinear in the Eqn. (3.50) can be rewritten as:

$$\begin{aligned} \mathbf{N}_a' \sigma^{\mu\nu} \varepsilon^{ab} \mathbf{N}_b' &= N_1' \sigma^{\mu\nu} N_2' - N_2' \sigma^{\mu\nu} N_1' \\ &= \frac{1}{2\sqrt{2}} \left[c\,\Psi_1 \sigma^{\mu\nu} \Psi_2 - i\sqrt{2}\,\nu\,\sigma^{\mu\nu}\Psi_2 - c\,\Psi_2 \sigma^{\mu\nu}\Psi_1 + i\sqrt{2}\,\Psi_2 \sigma^{\mu\nu}\nu \right]. \end{aligned} \tag{3.60}$$

Exploiting now the $\sigma^{\mu\nu}$ antisymmetry, for which $\chi\sigma^{\mu\nu}\phi = \frac{1}{2}(\chi\sigma^{\mu\nu}\phi - \phi\sigma^{\mu\nu}\chi)$, for any field χ, ϕ, we can write

$$N_1' \sigma^{\mu\nu} N_2' - N_2' \sigma^{\mu\nu} N_1' = \frac{1}{\sqrt{2}}\, c\,\Psi_2 \sigma^{\mu\nu}\Psi_1 + is\,\Psi_2\sigma^{\mu\nu}\nu\,. \tag{3.61}$$

To convert the entire term, we must promote the 2×2 tensor $\sigma^{\mu\nu}$ into its four-component equivalent. Since the Gamma matrices γ^μ are the four-component counterpart of the Pauli matrices σ^μ, it is natural to consider the 2×2 anti-symmetric tensor promoted into

$$\Sigma^{\mu\nu} \equiv \frac{i}{2}\left[\gamma^\mu, \gamma^\nu\right] = 2\begin{pmatrix} \sigma^{\mu\nu} & 0 \\ 0 & \bar{\sigma}^{\mu\nu} \end{pmatrix}, \tag{3.62}$$

where $\bar{\sigma}^{\mu\nu} \equiv (\sigma^{\mu\nu})^\dagger = (i/4)(\bar{\sigma}^\mu\sigma^\nu - \bar{\sigma}^\nu\sigma^\mu)$. Now, promoting the bilinear using the new antisymmetric tensor, we obtain

$$\overline{\mathbf{\Psi}}\Sigma^{\mu\nu}\mathbf{\Psi} = \left(\Psi_2, \Psi_1^\dagger\right)\Sigma^{\mu\nu}\begin{pmatrix}\Psi_1 \\ \Psi_2^\dagger\end{pmatrix} = \Psi_2\sigma^{\mu\nu}\Psi_1 + \Psi_1^\dagger\bar{\sigma}^{\mu\nu}\Psi_2^\dagger. \tag{3.63}$$

However, to complete the equivalence with the dimension-five operator only the first term on the RHS of the equation is expected. The second term, $\Psi_1^\dagger\bar{\sigma}^{\mu\nu}\Psi_2^\dagger$, is in fact expected to be produced by the Hermitian conjugate of the LHS. Therefore, an object is needed in the LHS, ensuring the disappearance of $\Psi_1^\dagger\sigma^{\mu\nu}\Psi_2^\dagger$ while still maintaining a four-component form. The solution is the 4×4 matrix $P_L \equiv (\mathbb{1} - \gamma^5)/2$, which projects a spinor onto its left-handed component, as

$$P_L\mathbf{\Psi} = \begin{pmatrix}\Psi_1 \\ 0\end{pmatrix}. \tag{3.64}$$

Therefore, the product $\Sigma^{\mu\nu}P_L$ returns exactly the combination required by the dipole operator. Finally, the Lagrangian reads:

$$\mathcal{L}_{dipole} = \frac{1}{2} d\left(-c\overline{\mathbf{\Psi}}\Sigma^{\mu\nu}P_L\mathbf{\Psi} + s\overline{\mathbf{\Psi}}\Sigma^{\mu\nu}\nu_L\right)B_{\mu\nu} + h.c.\,. \tag{3.65}$$

Similar reasoning can be applied to the more general three flavor case, defining the massless Dirac active neutrino fields as

$$\nu_{L\alpha} = \begin{pmatrix} \nu_\alpha \\ 0 \end{pmatrix}, \tag{3.66}$$

where $\alpha = (e, \mu, \tau)$ and recalling the relations in Eqn. (2.72). With those new fields definitions in terms of the mass eigenvalues, the Eqn. (3.61) becomes:

$$N_1'\sigma^{\mu\nu}N_2' - N_2'\sigma^{\mu\nu}N_1' = \frac{1}{2}\Big[c_e c_\mu c_\tau(\Psi_1\sigma^{\mu\nu}\Psi_2 - \Psi_2\sigma^{\mu\nu}\Psi_1) - \sqrt{2}i\big[c_\tau c_\mu s_e(\nu_e\sigma^{\mu\nu}\Psi_2 - \Psi_2\sigma^{\mu\nu}\nu_e) \\ c_\tau s_\mu(\nu_\mu\sigma^{\mu\nu}\Psi_2 - \Psi_2\sigma^{\mu\nu}\nu_\mu) + s_\tau(\nu_\tau\sigma^{\mu\nu}\Psi_2 - \Psi_2\sigma^{\mu\nu}\nu_\tau)\big]\Big], \tag{3.67}$$

which can be further simplified using again the antisymmetry property of $\sigma^{\mu\nu}$. In conclusion the dipole term reads:

$$\begin{aligned} \mathcal{L}_{dipole} = &-\frac{1}{2}dc_e c_\mu c_\tau \overline{\mathbf{\Psi}}\Sigma^{\mu\nu}P_L\mathbf{\Psi}B_{\mu\nu} \\ &+\frac{1}{2}d\overline{\mathbf{\Psi}}\Sigma^{\mu\nu}(s_\tau\nu_{L\tau} + c_\tau s_\mu\nu_{L\mu} + c_\tau c_\mu s_e\nu_{Le})B_{\mu\nu} + h.c.\,, \end{aligned} \tag{3.68}$$

where no dipole interaction is allowed between different active neutrino flavors, due to the structure of the unitary matrix U avoiding the presence of physical ν_α inside the sterile field N_2'. In fact, a different rotation would generate a $\nu_\alpha\sigma^{\mu\nu}\nu_\beta$ term, violating the lepton number by two units.

This result becomes of particular interest when the $U(1)_N$ breaking limit is considered. Requiring that $\mu_2 = 2\delta M_s$, in fact, the unitary matrix $U_{sterile}$ defined in Eqn (2.69) gets minimally modified [10]:

$$U_{sterile} = \frac{1}{\sqrt{2}}\begin{pmatrix} -i\sqrt{1+\delta} & \sqrt{1-\delta} \\ i\sqrt{1-\delta} & \sqrt{1+\delta} \end{pmatrix}, \tag{3.69}$$

exactly as done for $U(1)_N$ breaking limit of the single flavor case, and where δ is, again, the sterile mass splitting parameter. An important relation between the mixing angles can be extracted from here:

$$|\theta_{2\alpha}| = \sqrt{\frac{1-\delta}{1+\delta}}\,|\theta_{1\alpha}|. \tag{3.70}$$

Hence, this case assumes no degeneracy in the two sterile masses and the diagonal mass matrix reads $\mathcal{M}_{diag} = diag(0, 0, 0, (1-\delta)M_s, (1+\delta)M_s)$. Then, the two sterile primed fields relate to the mass eigenstates as:

$$\begin{aligned} N_1' &= \frac{1}{\sqrt{2}}c_e c_\mu c_\tau\left(N_2\sqrt{1-\delta} - iN_1\sqrt{1+\delta}\right) - c_\tau c_\mu s_e\nu_e - c_\tau s_\mu\nu_\mu - s_\tau\nu_\tau\,, \\ N_2' &= \frac{1}{\sqrt{2}}\left(N_2\sqrt{1+\delta} + iN_1\sqrt{1-\delta}\right), \end{aligned} \tag{3.71}$$

where, again, the N_2' field does not mix with the active physical sector, indicating that no transition dipole moments are generated by this symmetry breaking

pattern. This is not a general case, obviously, since a contribution to the active mass matrix is generally allowed. In conclusion, the dipole Lagrangian term assume the form:

$$\begin{aligned}\mathcal{L}_{dipole} = & -idc_e c_\mu c_\tau N_1 \sigma^{\mu\nu} N_2 B_{\mu\nu} \\ & + \frac{1}{\sqrt{2}} d\sqrt{1+\delta} N_2 \sigma^{\mu\nu}(c_\tau c_\mu s_e \nu_e + c_\tau s_\mu \nu_\mu + s_\tau \nu_\tau) B_{\mu\nu} + \\ & + \frac{i}{\sqrt{2}} d\sqrt{1-\delta} N_1 \sigma^{\mu\nu}(c_\tau c_\mu s_e \nu_e + c_\tau s_\mu \nu_\mu + s_\tau \nu_\tau) B_{\mu\nu} + h.c.\,,\end{aligned} \tag{3.72}$$

representing the starting point for the decay widths computation. Here, the terms have been carefully split to highlight the different processes induced by each one, namely $N_2 \to N_1 + \gamma$ for the first term and $N_{1,2} \to \nu_\alpha + \gamma$ for the other two terms. Note that those decay will not allow the production of a Z boson since we are considering sterile masses below 10 GeV, together with a small mass splitting.

3.6.4 Dipole Coupling Induced Radiative Decays

Introducing the dipole operator in the sterile Lagrangian, thus, opens a new and important portal for HNL decays, non-trivially tied to the mixing parameters. The need to deeply study those processes arises not only because the total decay width includes them, but also because it may be dominated by them. As said before, the radiative decays induced by the dipole operator are of two different types, sterile-sterile decays ($N_2 \to N_1 + \gamma$), completely independent of the mixing angles, and active-sterile decays ($N_{1,2} \to \nu_\alpha + \gamma$), where a dipole-mixing interplay takes place. The sterile-sterile decay $N_1 \to N_2 + \gamma$ is deliberately not included in this classification since the hierarchy $M_2 \geqslant M_1$ is assumed. The mass splitting parameter δ, in fact, governs the two sterile-sterile decays and it always forbids one of them depending on its sign or both in the $\delta \to 0$ limit. If it is the case, the two sterile neutrinos naturally rearrange in the Majorana four-component fermion $\mathbf{\Psi}$ and only two flavor-dependent active-sterile decays are allowed: $\mathbf{\Psi} \to \nu_\alpha + \gamma$ and $\overline{\mathbf{\Psi}} \to \nu_\alpha + \gamma$, which have the same decay width Γ. In the case of non-vanishing mass splitting and at leading order in the active-sterile mixing angles, the radiative decay widths induced by the electromagnetic portal are [10, 11]:

$$\begin{aligned}\Gamma(N_2 \to N_1 \gamma) &= \frac{|d|^2 \cos^2\theta_w}{8\pi} M_2^3 \left(1 - \frac{M_1^2}{M_2^2}\right)^3, \\ \Gamma(N_2 \to \nu_\alpha \gamma) &= \frac{M_2^3}{8\pi} \left| d \cos\theta_w \sqrt{\frac{1+\delta}{1-\delta}} \right|^2 |\theta_{2\alpha}|^2, \\ \Gamma(N_1 \to \nu_\alpha \gamma) &= \frac{M_1^3}{8\pi} \left| d \cos\theta_w \sqrt{\frac{1-\delta}{1+\delta}} \right|^2 |\theta_{1\alpha}|^2,\end{aligned} \tag{3.73}$$

where the $\cos\theta_w$ corresponds to the projection of the hypercharge field strength tensor $B_{\mu\nu}$ onto the electromagnetic tensor $F_{\mu\nu}$, and thus θ_w is the Weinberg

angle. Crucially, these decay widths are expected to be small since $|d|M_i \ll 1$, to maintain mathematical consistency with the considered effective theory. Furthermore, the small mass splitting adds another suppression factor to $\Gamma(N_2 \to N_1\gamma)$ through the $\left(1 - \frac{M_1^2}{M_2^2}\right)$ term, while the experimental constraints on the mixing angles are responsible for an additional suppression on the active-sterile widths. All those factors cause the decay widths to be small and, as a consequence, the sterile neutrinos are expected to be long-lived [10].

It is crucial to analyze the kinematic dependence of the radiative decay widths when comparing them to the mixing-induced Γ, to determine the dominant process in the phenomenology. From Eqn. (3.73) it is clear that both sterile-sterile and active-sterile decays scale asymptotically as M_i^3, while, on the contrary, the scaling of the mixing-induced decays depends on the mass range. In particular, for $M_i \gtrsim 1.6$ GeV the QCD non-perturbative regime breaks down and the three-body decay channels open and dominate the active-sterile mixing sector since at these energies free quarks are generated. The decay widths of processes like $N_i \to l_\alpha^- l_\beta^+ \nu_\beta$ or $N_i \to l_\alpha^- q_u \bar{q}_d$, presented in App. C of Ref. [10], show a clear scaling of M_i^5 [42]. It is this difference in kinematic behavior that causes the dipole portal to dominate for small masses ($M_i \lesssim 1.6$ GeV), while it becomes inevitably subdominant once the QCD perturbative regime introduces the faster scaling, assuming comparable coefficients for the dipole and the mixing portals. A more in-depth discussion on the QCD regimes is presented in Appendix A. The plot presented in Fig. 3.8 shows the decay widths for the dipole, μ-mixing and τ-mixing channels separately. Their computation has been carried out making use of the parameters described in Ref. [10], namely: $|\theta_{i\mu}| = 3 \times 10^{-4}$, $|\theta_{i\tau}| = 10^{-3}$ and the weak coupling case $d = 2.26 \times 10^{-6}$ GeV^{-1} has been considered. Although the reference mass scale considered is M_1, chosen for comparability, those decay widths are computed for N_2, so that also the $N_2 \to N_1\gamma$ decay is considered. The mass splitting parameter is then fixed to $\delta = 0.1$, making N_2 slightly heavier than N_1.

The detailed analysis of the decay widths raises a crucial question regarding the total lifetime τ_N of the sterile states N_i. In fact, their potential late decay can play a decisive role in the evolution of the Universe and it risks altering the final abundances of light nuclei predicted by Big Bang Nucleosynthesis (BBN), particularly the yields of helium and deuterium [12]. To ensure they decay before the onset of nucleosynthesis, a stringent upper bound on the sterile neutrino lifetime is imposed, typically requiring $\tau_N \lesssim 1$ s.

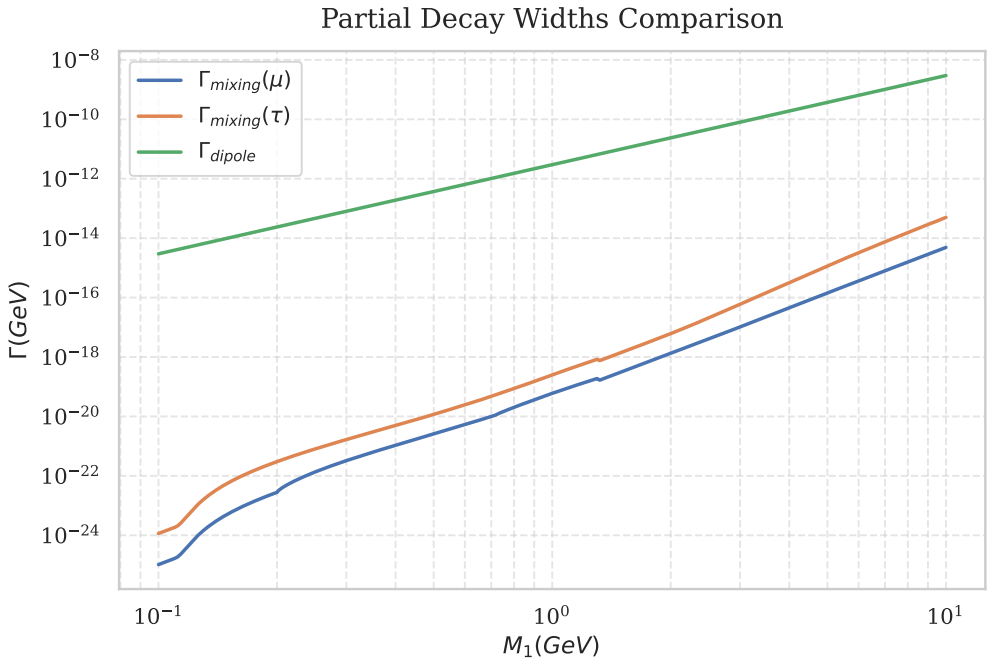


Figure 3.8: Comparison of the partial decay widths for the heavier sterile neutrino state (N_2) as a function of the reference mass M_1, fixing a mass splitting $\delta = 0.1$. A weakly-coupled scenario ($d = 2.26 \times 10^{-6}$ GeV^{-1}) has been considered for the total radiative decay width induced by the dipole operator. The mixing-induced decay widths consider the active-sterile mixing exclusively with the muon ($|\theta_{i\mu}| = 3 \times 10^{-4}$) and tau ($|\theta_{i\tau}| = 10^{-3}$) flavors, respectively. The distinct kinematic transition in the mixing-induced channels is visible around $M_1 \sim 1.6$ GeV, where the onset of three-body partonic decays shifts the scaling from M^3 to M^5. In this specific parameter space, the dipole portal dominates the total width across the entire mass spectrum shown.

Chapter 4

BBN Bounds

The detailed analysis of the decay widths shown in the previous chapter raises a crucial question regarding the total lifetime τ_N of the sterile states N_i. So far, the phenomenological impact of these decay times has been framed primarily within the context of terrestrial searches at colliders and high-intensity experiments, where macroscopic times of flight translate into displaced vertices or missing energy signatures [41]. However, a different bound emerges from the dynamics of the early Universe. If the sterile states N_i reach thermalization within the cosmological plasma due to strong coupling to SM, their relic number density won't be negligible anymore. Once the thermal bath cools down to temperatures $T \lesssim M_i$, those states decay into accessible SM particles, injecting high-energy particles out of thermal equilibrium. Their potential late decay plays a decisive role in the evolution of the Universe, especially if it happens during the Big Bang Nucleosynthesis (BBN). A perturbation in the nuclear chain reactions generated by these decays would cause a different production of light nuclei with respect to the observed yields [12]. These processes are:

$$\begin{aligned} N_2 &\to N_1 + \gamma, \qquad & N_i &\to \nu_\alpha + \ell_\beta^- + \ell_\beta^+, \\ N_i &\to \nu_\alpha + \gamma, \qquad & N_i &\to \ell_\alpha^- + q_u + \bar{q}_d. \end{aligned} \tag{4.1}$$

As we can see, both the dipole- and mixing-induced channels contribute to the injection of SM particles, namely leptons, quarks (and hadrons once the plasma cools down) and photons. However, up to now SM theoretical predictions and observational data agree on the primordial yields of Deuterium (D), Helium-4 (^4He), Helium-3 (^3He) and Lithium-7 (^7Li). Requiring this agreement to be maintained imposes a severe and robust bound on the sterile field lifetime τ_i, which must satisfy:

$$\tau_i \equiv \frac{\hbar}{\Gamma_{tot}(N_i)} \lesssim 1 \text{ s} , \tag{4.2}$$

ensuring their decay before the weak interactions freeze-out. Here $\Gamma_{tot}(N_i)$ represents the total decay width obtained by summing the contributions from all processes presented in Appendix A.

Sterile neutrinos stay in thermal equilibrium with the cosmological plasma as long as their interaction rate is greater than the Universe expansion rate:

$$\Gamma_{tot}(T) = \Gamma_{dip}(T) + \Gamma_{mix}(T) > H(T), \tag{4.3}$$

where $H(T)$ is the Hubble cosmological expansion rate and its temperature dependence is entirely defined by the first Friedmann equation for the radiation energy density[1]

$$\rho_{rad}(T) = \frac{\pi^2}{30} g_*(T) T^4. \tag{4.4}$$

The Hubble constant can then be derived [43]:

$$H(T) = \sqrt{\frac{8\pi G}{3}\rho_{rad}(T)} \simeq 1.66\sqrt{g_*(T)}\frac{T^2}{M_{Pl}}, \tag{4.5}$$

where $g_*(T)$ describes the total thermalized degrees of freedom and $M_{Pl} \simeq 1.22\times10^{19}$ GeV is the Planck mass. Therefore, thermal equilibrium is guaranteed if at least one channel between the mixing and the dipole satisfies the $\Gamma(T) \gg H(T)$ condition. This happens for different temperatures of the thermal bath due to their different scaling:

$$\begin{aligned} \frac{\Gamma_{dip}(T)}{H(T)} &\propto \frac{|d|^2 T^3}{T^2/M_{Pl}} \propto |d|^2 M_{Pl} T, \\ \frac{\Gamma_{mix}(T)}{H(T)} &\propto \frac{G_F^2 |\theta_{k\alpha}|^2 T^5}{T^2/M_{Pl}} \propto G_F^2 |\theta_{k\alpha}|^2 M_{Pl} T^3. \end{aligned} \tag{4.6}$$

It is clearly visible that a higher temperature is required for the dipole channel to hold the thermal equilibrium. So, while dipole-induced interactions are efficient only at extremely high temperatures, the mixing mechanism ensures production also at $T \sim$ few GeV [44]. In this scenario, the sterile neutrino number density is described by [43]:

$$n_i^{eq}(T) = \frac{g_i}{2\pi^2}\int_0^\infty \frac{p^2\, dp}{\exp\left(\frac{\sqrt{p^2+M_i^2}}{T}\right)+1}, \tag{4.7}$$

where the helicity degrees of freedom are $g_i = 2$ for each sterile state N_i. As the Universe expands, its temperature diminishes until the thermalization condition (4.3) holds no more and the sterile states decouple from the primordial plasma. Their freeze-out dynamics depends on the decoupling temperature T_{dec} and on their yield, defined as:

$$Y_i \equiv \frac{n_i}{s}, \tag{4.8}$$

where $s(T) = \frac{2\pi^2}{45} g_{*S}(T) T^3$ is the cosmological plasma entropy number density. Two different regimes for the yield arise from different possible decoupling temperatures. If $T_{dec} \gg M_i$, the HNLs decouple when still relativistic, causing their yield not to be suppressed, while instead if $T_{dec} \lesssim M_i$, sterile states are no longer relativistic during their decoupling, so the number density suffers an exponential Maxwell-Boltzmann suppression. This last case causes the freeze-out abundance to be significantly smaller with respect to the first regime. In either case, after the decoupling, the yield Y_i^{fo} remains constant until the time $t \sim \tau_i$ is reached. At this point sterile decays become important, causing the

[1] Here only radiation energy density ρ_{rad} is considered since the described processes happen during the radiation domination epoch.

injection of high energy SM particles mentioned above. The exact same reasoning applies to weak interactions, whose freeze-out happens, once again, when $\Gamma_{weak}(T_{fo}) \simeq H(T_{fo})$ is satisfied. The temperature of this event corresponds to $T_{fo} \simeq 0.8$ MeV, happening for times $t \simeq 1$ s, and it means that protons and neutrons are non-relativistic during their decoupling, as expected. From that moment neutrons, since unstable, start to decay ($\tau_n \simeq 880$ s) changing their frozen yield and the proton one during the subsequent cosmological phase, the BBN. Here, almost all neutrons get captured into the (extremely stable) ^4He nuclei, whose yield Y_p is therefore fixed by the mass ratio $(n/p)_{BBN}$ as [45]:

$$Y_p \simeq \frac{2(n/p)_{BBN}}{1 + (n/p)_{BBN}} \simeq 0.25\,. \tag{4.9}$$

This production mechanism is extremely sensitive to the plasma conditions during the whole process, going from the weak freeze-out to the end of BBN, estimated at $T \sim 0.1$ MeV and $t \sim 3$ min. For this reason it is essential that $\tau_i \lesssim 1$ s for the sterile neutrino model to be valid, or energy injection during this time window would alter significantly the light nuclei yields. On the contrary, a short lifetime for the sterile fields would ensure their SM particle production to be absorbed into the plasma and only the relativistic degrees of freedom would be affected. They are parametrized by the effective number of active neutrinos ΔN_{eff}, which must remain within the experimental limits imposed by the Cosmic Microwave Background (CMB) for the model to be consistent [5].

Late neutrino decays would happen inside a plasma no more capable of thermalizing efficiently their products. For example, the dipole-induced reaction $N_i \to \nu_\alpha \gamma$ generates photons with $E_\gamma \simeq M_i/2$ that will interact with the less energetic photons of the thermal bath. This would induce electron-positron pair production reactions, $\gamma + \gamma_{bath} \to e^+ + e^-$, that then would participate in inverse Compton scattering $e^\pm + \gamma_{bath} \to e^\pm + \gamma$. In this way a chain of processes is generated, injecting in the plasma secondary photons with a continuous and out of equilibrium energy spectrum distribution. The consequences of a non thermalized photon population become important when their energy is greater than the binding energy of the newly formed light nuclei. In fact, an inelastic scattering with those energetic photons would tear them apart in a reaction called photodisintegration. The nucleus most sensitive to this mechanism is the Deuterium, due to its low binding energy of just $E_D \simeq 2.22$ MeV. Its yield is strictly constrained by the baryon-photon ratio and the process $\gamma_{non\,thermal}$ + D $\to p + n$ would reduce it significantly. At a slightly higher energy ($E_{He4} \simeq$ 19.8 MeV), the reaction ^{4}He $\to^3$ H+p would give similar results for the Helium-4 yield [13].

A complementary role is played by the mixing-induced channel which introduces non-thermal mesons, in particular pions $\pi^{0,\pm}$, and alters the nucleon population via reactions of the type [13]:

$$p + \pi^- \to n + \pi^0. \tag{4.10}$$

This shifting of the neutron-proton ratio n/p towards the neutral side generates an important increase in the ^4He production, violating the observational limits.

4.1 Phenomenological Exclusions in the Parameter Space

The cosmological requirement of a short lifetime for sterile neutrinos ($\tau_i \lesssim 1$ s) translates into a constraint for the theory parameters. In particular, the excluded regions can be mapped onto the parameter space (M_1, θ_1), treating the dipole coefficient d and the mass splitting δ as free parameters. This bound effectively selects regions of the parameter space characterized by stronger interactions, reducing therefore the lifetime. In fact, excessively small mixing angles or dipole coefficients would reduce the interaction rate and, in the same way, a small sterile mass would allow fewer decay channels. Hence, those cases should be discarded. The plots in Fig. 4.1 clearly illustrate this behavior. The lifetime of the lighter sterile neutrino has been computed assuming mixing with a single active flavor at a time, $\theta_{1\mu}$ or $\theta_{1\tau}$, and closing the interaction channels with the other two lepton flavors. Switching on different mixing should change the threshold in the decay channel and, in general, the behavior of the decay itself, but no significant difference is visible when comparing the $\theta_{1\mu}$ and $\theta_{1\tau}$ panels. This shows that the prediction is in practice only very weakly dependent on which mixing is turned on. Instead, the lifetime is governed mainly by the relative importance of the dipole contribution over the mixing one in the decay channels. Choosing which lepton flavor to mix with the HNLs does not weight much on the BBN prediction, which are therefore robust against this specific choice.

For each plot, many dipole coefficients are chosen by varying the New Physics coupling g_* and its cutoff mass scale m_*, from the parametrization

$$d = \frac{g_Y}{16\pi^2}\frac{g_*^2}{m_*}. \tag{4.11}$$

As can be seen, larger dipole coefficients correspond to smaller excluded regions. In fact, as the interactions with the SM gets stronger, the decay probabilities grow and their lifetime shortens. In this way, higher dipole coefficients guarantee to satisfy the BBN request even at small mixing angles. The values chosen for the dipole coefficient come from plausible values of its unknown parameters. The New Physics coupling is assumed to be $g_* = 1$ or $g_* = 4\pi$, corresponding to the weak coupling regime and the limit for the perturbative regime, respectively. Instead, the UV completion mass scale has been fixed to $m_* = 1$ TeV, 10 TeV, 100 TeV. The first mass choice comes from the expected future observations due to the LHC limits, then heavier scales have subsequently been considered. It is interesting to observe how the case of vanishing dipole ($d \to 0$) perfectly overlaps with the heaviest New Physics scenario, implying that for weak NP coupling and $m_* \simeq 100$ TeV the contribution of the dimension-five operator becomes negligible and has no effects on the BBN bound. In this limit the sterile lifetime depends to excellent approximation only on the mixing parameter

Two different cases for the mass parameter splitting have been considered: $\delta = 0$ and $\delta = 0.5$. In the first scenario the two neutrinos $N_{1,2}$ have the same mass, forbidding the $N_2 \to N_1\gamma$ decay channel. On the contrary, as soon as $\delta \neq 0$ this channel is allowed and proportional to δ itself as explained in Sec. 3.6.4. Therefore, the second scenario includes the conversion $N_2 \to N_1$, enhanc-

ing the lighter sterile neutrino population. This mechanism requires stronger interactions (and thus larger mixing parameters) to keep the cosmological constraint satisfied. Obviously, since this process is governed by the dipole operator, smaller coefficients lead to a negligible or even vanishing effect. Lastly, every plot in Fig. 4.1 presents a bump whenever a new decay channel opens. Particularly evident is the drop at $M_1 \simeq 140$ MeV, corresponding to the opening of pion production. A small rise is present at $M_1 = 1.6$ GeV, where the QCD perturbative regime is considered and free quarks are generated. In panel **(a)**, the bump at $\simeq 210$ MeV meets the three-body decay channel where a $\mu^+\mu^-$ is produced. On the contrary, the shallower slope after $\simeq 210$ MeV in panel **(c)** is due to the τ mass which delays the opening of leptonic production channels only for larger masses.

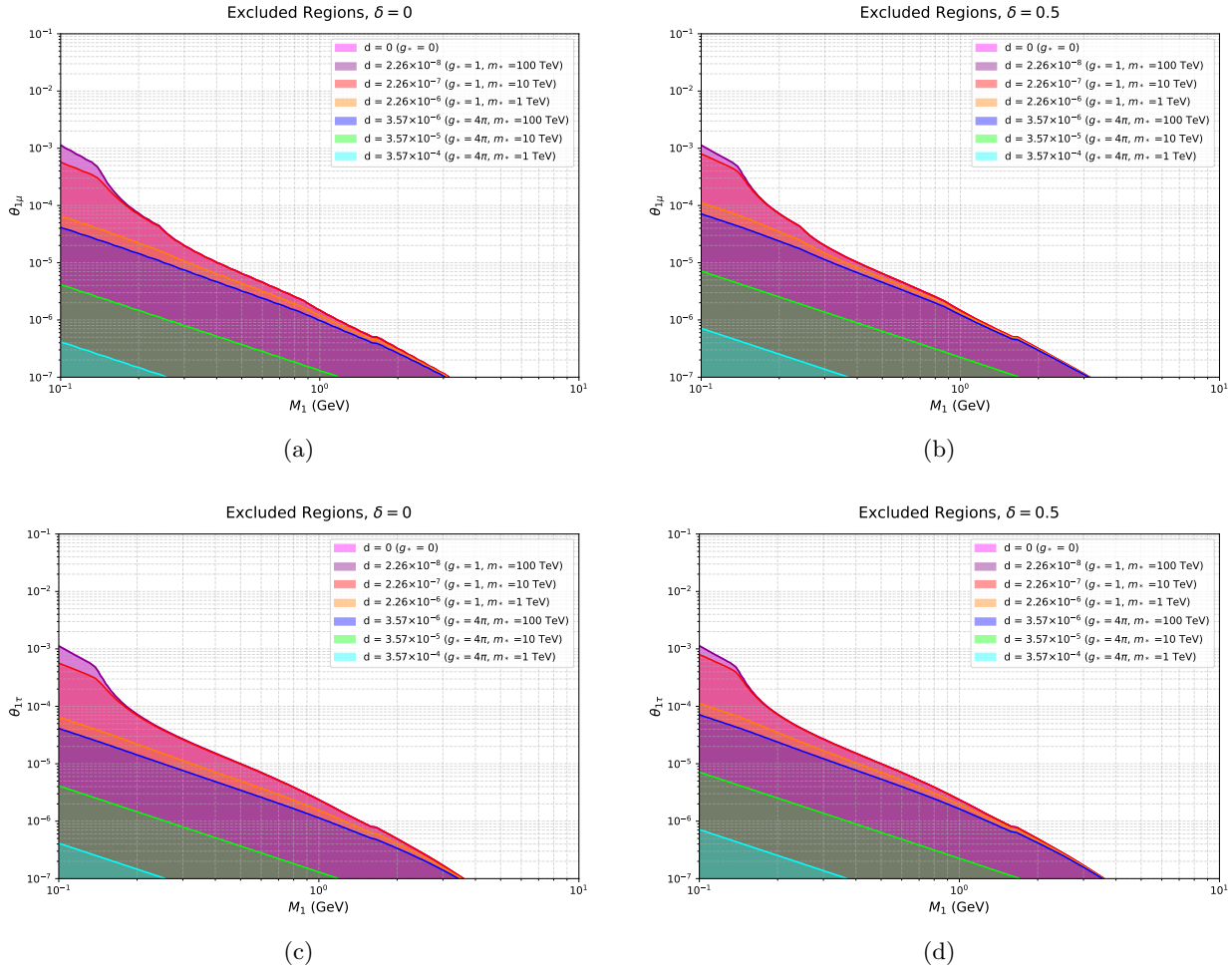


Figure 4.1: BBN excluded regions in the (M_1, θ_1) plane for the minimal νSMEFT scenario. The colored shaded areas denote the parameter space where the sterile neutrinos are excessively long-lived ($\tau \gtrsim 1$ s), thus spoiling the successful predictions of Big Bang Nucleosynthesis. The contours demonstrate how the inclusion of the sterile dipole interaction (parameterized by different UV scales m_* and couplings g_*) enhances the total decay width, progressively shrinking the excluded region and allowing for smaller values of the active-sterile mixing angle.

Chapter 5

Conclusions

Throughout the entire discussion the phenomenological implications of Heavy Neutral Leptons (HNLs) have been explored, focusing within the mass range $0.1 \text{ GeV} \leqslant M_N \leqslant 10 \text{ GeV}$. The effects generated by both the active-sterile mixing via the Minimal Seesaw Model and the contributions of the dimension-five dipole operator have been considered. Crucially, due to the interaction of the sterile neutrino with the hypercharge gauge boson, the presence of the dipole operator $\mathcal{O}_{NB}^{(5)}$ introduces a mixing-independent portal allowing for radiative decays. Our primary objective was to assess how this higher-dimensional interaction alters the decay dynamics of sterile neutrinos and how cosmological observations constrain the viable parameter space of this effective theory. In particular, the Big Bang Nucleosynthesis (BBN) requirement of $\tau_i \lesssim 1$ s was implemented into the model since it represents the most robust bound. A thorough analysis of the sterile neutrino decay widths revealed a significant interplay between the mixing-induced and dipole-induced channels. We observed a fundamental kinematic transition that dictates the dominant decay mechanism across different mass regimes. For lighter sterile masses ($M_i \lesssim 1.6$ GeV), the dipole-mediated radiative decays ($N_2 \to N_1\gamma$ and $N_i \to \nu_\alpha\gamma$) often dominate the total width, owing to their M^3 scaling. Conversely, as the mass exceeds the perturbative QCD threshold, the phase space for mixing-induced three-body partonic decays opens up [42]. The steeper M^5 scaling of these weak interactions may rapidly overtake the dipole contribution, causing the mixing portal to govern the sterile lifetime at higher energies, as long as the dipole coefficient is comparable with the mixing angles.

The injection of non-thermal particles in the primordial plasma would, in general, spoil the successful predictions of BBN. This can be avoided requiring the RH neutrinos to have a sufficiently short lifetime, $\tau_i \lesssim 1$ s, in such a way that most of their decays happens before the beginning of nucleosynthesis. The constraint generated by this requirement can be mapped onto the (M_1, θ_1) parameter space, demonstrating how the radiative decays can affect the allowed regions by varying the dipole coefficient. The BBN excluded regions shrink due to the dipole shortening the HNL lifetime, opening up parameter space that would be otherwise ruled out in a purely mixing-driven scenario. Smaller mixing angles are therefore allowed in a strong New Physics coupling scenario. The mass splitting parameter δ, plays instead an opposite role, governing the radiative transition $N_2 \to N_1\gamma$ and enhancing in this way the light sterile population.

This process must be balanced by larger mixing angles to satisfy the BBN bound and ensure a sufficiently rapid depletion of the N_1 density. This demonstrates that the specific mass hierarchy within the sterile sector non-trivially impacts the cosmological constraints.

In conclusion, the presence of dimension-five dipole interactions relaxes the lower bounds on the active-sterile mixing angles imposed by BBN. This theoretical finding has profound implications for future experimental searches. Since HNLs with dipole couplings can be cosmologically safe even with heavily suppressed mixing parameters, they might easily evade detection at current intensity frontier experiments. Therefore, the development of future high-luminosity colliders and dedicated long-lived particle detectors, sensitive to displaced vertices and missing energy signatures, will be paramount to probe the remaining open regions of the SMEFT parameter space and either confirm or rule out the existence of these elusive sterile states [37, 41].

Appendix A

Decay width computation

Here a more in-depth study of the radiative and mixing-induced processes is presented to complete the discussion on Sec. 3.6.4 and to support it with explicit computations. The following Appendix is divided into two sections. Firstly, all phenomenologically important processes are shown and discussed, to pave the way for Chapter 4. In the second section, instead, the explicit computation of a decay width is carried out step by step.

A.1 Decays Channels for Sterile Neutrinos

The present section is intended to serve as a small compendium where all processes affecting the sterile sector are summarized and the total decay width is built. The single decay widths for each process listed here can be found in App. C of Ref. [10].

While radiative decays and their widths Γ are shown in Sec. 3.6.4, and are divided into only two classes of processes, the mixing-induced reactions between the sterile HNLs and the SM form a more complex and wide panorama. In fact, by substituting the flavor eigenstates ν'_α with their definition in terms of the mass eigenstate fields, described by Eqn. (2.72), new weak interactions are generated between sterile states and fermions within the renormalizable SM Lagrangian. Those new couplings therefore allow the production of not only active neutrinos and charged leptons, which couple directly to HNLs, but also with quarks through a weak boson mediator ($W^\pm, Z$), which is also generated directly in the sterile decay vertex. Note that a coupling vertex including a photon is not allowed at tree-level due to the vanishing electric charge of neutrinos. Hence, sterile decays can generate three leptons as final product or one lepton (charged or not) together with a quark and an antiquark. Crucially, when the quark production is considered, it becomes important to define the energy at which the decay happens and so the mass of the unstable particle. This attention to the sterile mass is not required for the determination of the channels closure, for which a Heaviside Theta is already implemented, but because of the limited applicability range of the perturbative QCD regime. Indeed, for decays involving small sterile masses, the final quarks do not have enough energy to be produced free from gluon exchange among them and must then be considered as a single particle, a meson. This limit can be assumed to be precise, defining

two different decay regimes depending on whether $M_i \leqslant 1.6$ GeV, where QCD non-perturbative regime must be used, or $M_i > 1.6$ GeV [10, 42].

The sterile neutrino decay channels in the QCD non-perturbative regime are, therefore, the following:

$$\begin{aligned} N_i &\to l_\alpha^- P^+, \\ N_i &\to \nu_\alpha P^0, \\ N_i &\to l_\alpha^- V^+, \\ N_i &\to \nu_\alpha V^0, \end{aligned} \tag{A.1}$$

where $P^{0,+}, V^{0,+}$ are the pseudoscalar and vector mesons, respectively, and their charge depends on the lepton charge they are coupled to. The list of the considered mesons for the total decay width is shown in the Table A.1, no other mesons are considered due to their heavy masses preventing their production within the mass range considered for HNLs.

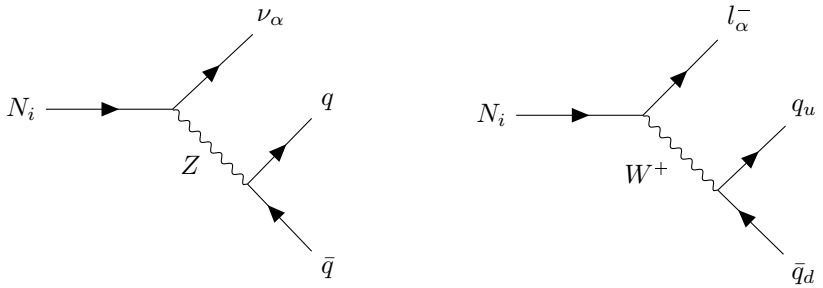
V^+ **mesons**	P^+ **mesons**	P^0 **mesons**	V^0 **mesons**
K^{*+}	π^+	π^0	ρ^0
D^{*+}	K^+	η_c	ω
D_s^{*+}	D^+	η	ϕ
ρ^+	D_s^+	η'	J/ψ
	B^+		Υ
	B_c^+		

Table A.1: List of light mesons considered for the computation of the total decay widths of heavy sterile neutrinos (N_i), categorized according to their spin and electric charge: charged vector (V^+), charged pseudoscalar (P^+), neutral pseudoscalar (P^0), and neutral vector (V^0) mesons. These hadronic final states define the decay channels accessible within the non-perturbative QCD regime, which is strictly applied for sterile neutrino masses $M_i \leqslant 1.6$ GeV. In this energy domain, the produced quark pairs lack sufficient kinetic energy to propagate as free particles and consequently hadronize into single mesons. Mesons heavier than those listed are excluded from the analysis, as their production is kinematically forbidden within the mass range considered for the Heavy Neutral Leptons (HNLs).

For similar reasons, in the QCD perturbative regime the decays into heavy quarks (c, b, t) are not considered due to their phase space suppression. The contributing decays in this limit are:

$$\begin{aligned} N_i &\to l_\alpha^- u\bar{d}, \\ N_i &\to l_\alpha^- u\bar{s}, \\ N_i &\to \nu_\alpha u\bar{u}, \\ N_i &\to \nu_\alpha d\bar{d}, \\ N_i &\to \nu_\alpha s\bar{s}. \end{aligned} \tag{A.2}$$

All those processes, for both the perturbative and non-perturbative regimes, are presented in Fig. A.1.



(a) $N_i \to \nu_\alpha q\bar{q}$ **(b)** $N_i \to l_\alpha^- q_u \bar{q}_d$

Figure A.1: Tree-level Feynman diagrams for the three-body weak decays of the heavy sterile neutrino N_i. (a) Weak neutral-current (NC) decay $N_i \to \nu_\alpha q\bar{q}$, mediated by the exchange of a virtual Z boson. The interaction at the primary vertex is induced by the active-sterile mixing parameter $\theta_{i\alpha}$ coupling the sterile state to the Standard Model neutral current. (b) Weak charged-current (CC) decay $N_i \to l_\alpha^- q_u \bar{q}_d$, mediated by the exchange of a virtual W^+ boson. The N_i-l_α^--W^+ vertex is governed by the same leptonic mixing parameter $\theta_{i\alpha}$. For sterile neutrino masses below the perturbative QCD scale ($M_i \lesssim 1.6$ GeV), the final-state quark pairs ($q\bar{q}$ and $q_u\bar{q}_d$) hadronize into single pseudoscalar or vector mesons (e.g., P^0, V^0 or $P^\pm, V^\pm$), which are described non-perturbatively via their respective hadronic decay constants [46].

Finally, the three-body decay channels are [10]:

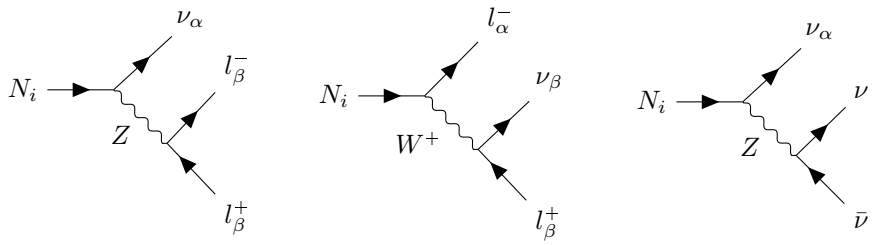
$$\begin{aligned} N_i &\to l_\alpha^- l_\beta^+ \nu_\beta, \\ N_i &\to \nu_\alpha l_\beta^+ l_\beta^-, \\ N_i &\to \nu_\alpha \nu \bar{\nu}, \end{aligned} \tag{A.3}$$

where $\alpha, \beta = (e, \mu, \tau)$ are the flavor indices. Those decays, are shown and explained in Fig. A.2.

The total decay width will depend on the considered regime. To build it, it is enough to sum all the contributions from the processes present in the chosen regime, paying attention to two important details. Firstly, the Heaviside step function must be included as a multiplicative factor to each Γ to ensure the channels opens only when the condition $M_i \geqslant \sum_f m_f$ is met. Secondly, a factor of 2 must be taken into account when the considered process admits its Hermitian conjugated counterpart, such as the charged meson decays or $N_i \to l_\alpha^- u\bar{d}$.

A.2 Interference Decay Width Computation

As anticipated, in this section, we present the explicit computation of $\Gamma(N_i \to \nu_\alpha V^0)$ decay width. This process has been chosen because it exhibits an interference effect between the mixing- and dipole-induced decays takes place. As shown in Fig. A.1(a) the process converts a sterile neutrino N_i into an active



(a) $N_i \to \nu_\alpha l^+_\beta l^-_\beta$ **(b)** $N_i \to l^-_\alpha l^+_\beta \nu_\beta$ **(c)** $N_i \to \nu_\alpha \nu \bar{\nu}$

Figure A.2: Tree-level Feynman diagrams for the three-body purely leptonic decays of the heavy sterile neutrino N_i, induced by the active-sterile mixing. (a) Decay into an active neutrino and a charged lepton pair. This process is primarily mediated by a neutral-current (NC) Z exchange (if $\alpha = \beta$, an additional charged-current $W^\pm$ exchange diagram contributes to the amplitude). (b) Decay into two charged leptons of different flavours ($\alpha \neq \beta$) and a neutrino, mediated strictly by a charged-current (CC) W^+ exchange. (c) Fully invisible decay into three active neutrinos, mediated by a NC Z boson.

neutrino ν_α and a virtual Z boson. Subsequently, the gauge boson couples to a quark pair, the neutral meson V^0. However, mesons of this kind can be produced by any neutral mediator, which includes also the photon γ, naturally generated in the dipole-induced decay $N_i \to \nu_\alpha \gamma$ shown in Fig. 3.7. This forces us to include both contributions when computing the total amplitude of the process, leading to an interference effect between the two:

$$\begin{aligned} |\mathcal{A}_{tot}|^2 &= |\mathcal{A}_{mixing} + \mathcal{A}_{dipole}|^2 \\ &= |\mathcal{A}_{mixing}|^2 + |\mathcal{A}_{dipole}|^2 + \mathcal{A}_{mixing}\mathcal{A}^\dagger_{dipole} + \mathcal{A}_{dipole}\mathcal{A}^\dagger_{mixing}. \end{aligned} \tag{A.4}$$

To completely understand the implications of the interference effect, the single process amplitudes will be derived separately, each starting from the Lagrangian terms of interest to us.

The dimension-five operator is governed by the Lagrangian in Eqn. (3.72), which needs to be converted into its four-components form. To do so, in the following will be used the definition of the mixing angles and, since they are assumed to be small, an approximation for their cosine will be taken:

$$\begin{aligned} \theta_{1\alpha} &= \frac{i}{\sqrt{2}} s_\alpha \sqrt{1+\delta}, \\ \theta_{2\alpha} &= \frac{1}{\sqrt{2}} s_\alpha \sqrt{1-\delta}, \\ c_\alpha &\sim 1. \end{aligned} \tag{A.5}$$

Then, omitting the first term of Eqn. (3.72), since it governs the $N_2 \to N_1 \gamma$ decay, the Lagrangian can be rewritten as:

$$\begin{aligned} \mathcal{L}_{dipole} &= id\theta_{2\alpha} N_1 \sigma^{\mu\nu} \nu_\alpha F_{\mu\nu} \cos\theta_w - id\theta_{1\alpha} N_2 \sigma^{\mu\nu} \nu_\alpha F_{\mu\nu} \cos\theta_w + h.c. \\ &= \pm\, id\theta_{j\alpha} N_i \sigma^{\mu\nu} \nu_\alpha F_{\mu\nu} \cos\theta_w + h.c. \end{aligned} \tag{A.6}$$

where $F_{\mu\nu}\cos\theta_w$ comes again from the projection of the $B_{\mu\nu}$ hypercharge field strength and therefore θ_w is the Weinberg angle. It is important to notice how the mixing angle carries a sterile flavor index $j \neq i$ and the sign of the term depends on the index i of the HNL. Passing now to the four-component notation and writing the Hermitian conjugate term explicitly, the Lagrangian reads:

$$\begin{aligned}\mathcal{L}_{dipole} &= \left[\pm i\frac{d}{2}\theta_{j\alpha}\overline{\boldsymbol{N}}_i\Sigma^{\mu\nu}P_L\boldsymbol{\nu}_\alpha \mp i\frac{d^*}{2}\theta^*_{j\alpha}\bar{\boldsymbol{\nu}}_\alpha\Sigma^{\mu\nu}P_R\boldsymbol{N}_i\right]F_{\mu\nu}\cos\theta_w \\ &= \pm\, i\cos\theta_w\overline{\boldsymbol{\nu}}_\alpha\Sigma^{\mu\nu}(d\,\theta_{j\alpha}P_L + d^*\theta^*_{j\alpha}P_R)\boldsymbol{N}_i\partial_\mu A_\nu\,,\end{aligned} \tag{A.7}$$

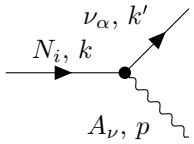
where $\boldsymbol{N}_i, \boldsymbol{\nu}_\alpha$ are the Majorana four-spinors, defined as $\boldsymbol{N}_i = \begin{pmatrix} N_i \\ N_i^\dagger \end{pmatrix}$, following the procedure described in Ref. [16] and $\Sigma^{\mu\nu} = i[\gamma^\mu, \gamma^\nu]/4$ is the four-dimensional antisymmetric tensor. The left- and right-handed projectors $P_{L,R} = (1 \pm \gamma^5)/2$ appear in the new notation to maintain the same Lagrangian structure as explained in Sec. 3.6.3. The second step is derived by swapping the active neutrino field with the sterile one. Due to the Grassmann algebra, the operation consists in:

$$\bar{\boldsymbol{\nu}}_\alpha\Sigma^{\mu\nu}P_L\boldsymbol{N}_i = -\overline{\boldsymbol{N}}_i\Sigma^{\mu\nu}P_L\boldsymbol{\nu}_\alpha. \tag{A.8}$$

Finally, let us notice the appearance of the photon field A_ν from the electromagnetic field strength tensor, for which its antisymmetry property has been used:

$$\Sigma^{\mu\nu}F_{\mu\nu} = 2\Sigma^{\mu\nu}\partial_\mu A_\nu, \tag{A.9}$$

where the photon momentum p_μ emerges from the partial derivative term in the momentum space and will correspond to the meson momentum. The Feynman vertex rule from the Lagrangian (A.7) is then:



$$= \pm i\cos\theta_w\Sigma^{\mu\nu}(d\,\theta_{j\alpha}P_R + d^*\theta^*_{j\alpha}P_L)p_\mu, \tag{A.10}$$

and the photon propagator will enter in the amplitude as:

$$-\frac{i\eta_{\nu\lambda}}{p^2}\,. \tag{A.11}$$

where $\eta_{\nu\lambda} = diag(1, -1, -1, -1)$.

Describing the meson vertex through the interaction Lagrangian is more complex than the sterile vertex, due to the quarks non-perturbative regime. The photon couples to the electromagnetic current through

$$\mathcal{L}_{\rm EM} = -eA_\mu J^\mu_{\rm EM} = -e\sum_q Q_q\bar{q}\gamma^\mu q A_\mu, \tag{A.12}$$

where Q_q is the quark charge in units of the electron charge e and q represents the quark field. Hence, the vertex Feynman rule for this term yields $-ie$. For the meson contribution in the process amplitude, instead, what matters are the

matrix elements of the photon current J^{μ}_{EM} between the vacuum and the final hadron state

$$\langle 0|J^{\mu}_{EM}|V^0(p)\rangle = i\frac{Q_q f_V m_V}{\sqrt{2}}\varepsilon^{\mu}(p), \tag{A.13}$$

where f_V is the specific decay constant of the vector meson[1], m_V its mass and ε^{μ} its polarisation vector, which depends on the photon momentum itself due to momentum conservation. In conclusion, the amplitude describing the sterile neutrino decay through the dipole portal reads:

$$\begin{aligned} i\mathcal{A}_{dipole} = \pm i\cos\theta_w\, \bar{u}^s_a(k')\left[\Sigma^{\mu\nu}(d^*\theta^*_{j\alpha}P_R + d\theta_{j\alpha}P_L)\right]_{ab} u^r_b(k) \\ \frac{(-ip_\mu\eta_{\nu\sigma})}{p^2}\,\frac{e}{\sqrt{2}}Q_q f_V m_V \varepsilon^{\lambda}_{\sigma}(p), \end{aligned} \tag{A.14}$$

where Greek letters are used for Lorentz indices and Latin letters are used for spinor indices, while α and j still refer to the flavor of the active and sterile sector respectively. The superscripts s, r, λ on the neutrino spinor fields u or ε define their polarisation. It follows that, after averaging the initial spins and summing over all possible final spins and polarisations,

$$\begin{aligned} |\mathcal{A}_{dipole}|^2 = +\frac{1}{2}\cos^2\theta_w\frac{e^2}{2}Q_V^2 f_V^2 m_V^2\left(-\eta_{\nu\beta} + \frac{p^\nu p^\beta}{m_V^2}\right)\left(\frac{p_\mu p_\alpha}{p^4}\right) \\ \mathrm{Tr}\left[(\not{k} + M_i)\Sigma^{\alpha\beta}(d\theta_{j\alpha}P_L + d^*\theta^*_{j\alpha}P_R)(\not{k}')\Sigma^{\mu\nu}(d\theta_{j\alpha}P_L + d^*\theta^*_{j\alpha}P_R)\right]. \end{aligned} \tag{A.15}$$

Here, we will remember that the sum over all possible spins or polarisations is given by

$$\begin{aligned} \sum_s u^s_a(k)\bar{u}^s_b(k) = (\not{k} + m)_{ab}, \\ \sum_\lambda \varepsilon^{\lambda}_{\alpha}(p)\varepsilon^{*\lambda}_{\beta}(p) = -\eta_{\alpha\beta} + \frac{p_\alpha p_\beta}{m_V^2}. \end{aligned} \tag{A.16}$$

Computing the trace of the spinor structure and inserting the momentum conservation laws:

$$\begin{aligned} M_i = |\vec{k}'| + E_V = |\vec{k}'| + \sqrt{m_V^2 + |\vec{p}|^2}, \\ \vec{k} = \vec{k}' + \vec{p} = 0, \end{aligned} \quad \text{where} \quad \begin{aligned} k &\equiv (M_i, 0), \\ k' &\equiv (|\vec{k}'|, \vec{k}'), \\ p &\equiv (E_V, \vec{p}), \end{aligned} \tag{A.17}$$

the squared amplitude reduces to

$$|\mathcal{A}_{dipole}|^2 = |d|^2 e^2 Q_V^2 f_V^2 \cos^2\theta_w |\theta_{j\alpha}|^2 \frac{M_i^2 - m_V^2}{2}\left(1 + 2\frac{M_i^2}{m_V^2}\right). \tag{A.18}$$

The same process can be applied for the mixing-induced term. The Lagrangian governing the sterile decay vector reads:

$$\mathcal{L}_{mixing} = \frac{g}{2\cos\theta_w} Z_\mu \overline{\boldsymbol{\nu}}_\alpha \gamma^\mu (\theta^*_{\alpha i}P_L - \theta_{\alpha i}P_R)\boldsymbol{N}_i\,, \tag{A.19}$$

[1]Numerical values for f_V can be found in Ref. [47, 48].

where g is the weak coupling. Note that, unlike before, the sterile flavor indices of the mixing angles and the HNL field now coincide. The resulting Feynman rules are therefore:

$$\begin{aligned} \text{Lepton vertex:} \qquad & i\frac{g}{2\cos\theta_w}\gamma^\mu(\theta^*_{\alpha i}P_L - \theta_{\alpha i}P_R), \\ \text{Boson propagator:} \qquad & \frac{1}{M_Z^2}, \\ \text{Quark vertex:} \qquad & i\frac{g}{2\cos\theta_w}. \end{aligned} \tag{A.20}$$

The computation of the meson contribution is derived in the same way as before. The neutral current (NC) now reads:

$$J_Z^\mu = \sum_q \left[\bar{q}\gamma^\mu(T_L^3 - 2Q_q\sin^2\theta_w)q - \bar{q}\gamma^\mu\gamma_5 T_L^3 q\right], \tag{A.21}$$

where T_L^3 corresponds to the eigenvalue of the third $SU(2)_L$ generator. The matrix element of the NC evaluated between the vacuum and the final states is:

$$\langle 0|J_Z^\mu|V^0(p)\rangle = i\frac{\kappa_V f_V m_V}{\sqrt{2}}\varepsilon^\mu(p), \tag{A.22}$$

where the κ_V factor takes into account that the current associated with each vector-meson appears into the Z current J_Z^μ with a specific coefficient [10]. Putting everything together we get:

$$i\mathcal{A}_{mixing} = -iG_F\bar{u}_a^s(k')\left[\gamma_\mu(\theta^*_{\alpha i}P_L - \theta_{\alpha i}P_R)\right]_{ab}u_b^r(k)\kappa_V f_V m_V \varepsilon_\mu^\lambda(p), \tag{A.23}$$

where the Fermi constant $G_F = \frac{\sqrt{2}}{8}\frac{g^2}{M_Z^2\cos^2\theta_w}$ has been introduced.

Concluding, the square modulus of the mixing amplitude reads:

$$|\mathcal{A}_{mixing}|^2 = 2G_F^2|\theta_{\alpha i}|^2\kappa_V^2 f_V^2 m_V^2|k'|M_i\left(\frac{M_i^2}{m_V^2}+2\right), \tag{A.24}$$

and the interference contribution yields:

$$\begin{aligned} A_{mixing}A^\dagger_{dipole} &= \mp iG_F\cos\theta_w\frac{i\,p_\alpha}{2\sqrt{2}p^2}eQ_V f_V^2 m_V^2\kappa_V\left(-\eta_{\mu\beta}+\frac{p_\mu p_\beta}{m_V^2}\right) \\ &\quad \mathrm{Tr}\left[(\not{k}')\gamma_\mu(\theta^*_{\alpha i}P_L - \theta_{\alpha i}P_R)(\not{k}+M_i)\Sigma^{\alpha\beta}(d\theta_{j\alpha}P_L + d^*\theta^*_{j\alpha}P_R)\right] \\ &= \mp G_F d^*\cos\theta_w\frac{eQ_V}{\sqrt{2}}f_V^2\kappa_V\cdot 6M_i^2|k'|\theta_i\theta_j^*. \end{aligned} \tag{A.25}$$

To avoid tedious computation, the second interference term can be determined just by observing that

$$A_{dipole}A^\dagger_{mixing} = \left(A_{mixing}A^\dagger_{dipole}\right)^\dagger. \tag{A.26}$$

Now that the expression for $|\mathcal{A}_{tot}|^2$ has been defined, computing the decay width Γ for the process is only an algebraic problem. In fact it is sufficient to substitute the amplitude in Eqn (A.4) into the decay width formula

$$\Gamma(N_i \to \nu_\alpha V^0) = \frac{|\vec{k}'|}{8\pi M_i^2}|\mathcal{A}_{tot}|^2. \tag{A.27}$$

Let us emphasize that no sterile flavor index has yet been defined. In order to find precise expressions for the $N_{1,2}$ decays the relations between the mixing angles have to be considered, shown in Eqn (A.5).

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